# INSTITUTIONS

OF ASTRONOMICAL

# CALCULATIONS.

PART II.

CONTAINING THE

## ASTRONOMY and GEOGRAPHY

OF

# TRANSITS,

AND THE

## PRINCIPLES of CALCULATION,

Illuſtrated and applied to the enſuing

TRANSIT of VENUS in 1769;

WITH THE

CONSTRUCTION and USE of a TRANSIT GLOBE

FOR THAT PURPOSE.

EMBELLISHED with THREE large COPPER PLATES,

AND A

MAP of the Great SOUTH SEA.

By B. MARTIN.

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# PREFACE.

*IN the first Part of this third Volume of Institutes having treated largely on the Doctrine of Solar and Lunar* ECLIPSES, *with the Rationale and Method of calculating the same from the best* TABLES *extant (which are given at the End thereof); I judged the Theory and astronomical Calculations of a* planetary TRANSIT *would very naturally follow that Subject; a Transit being in itself really a* solar Eclipse, *only in a smaller Degree. Besides, not only the* Curiosity *but* Importance *of such a Branch of Astronomy, so essentially necessary to the Perfection of the whole Science, could by no Means be omitted in a course of Institutions of this Kind. Add to this, that the Doctrine of* TRANSITS *is a recent Topic, never mentioned by former Astronomers, and but very slightly touched upon by the Moderns; the* Phænomenon *having been never but once publickly observed; and those Observations made in so imperfect a Manner, as to leave us rather under more Uncertainty about the* Solar Parallax *than we were before; and therefore, an Attempt to elucidate so intricate and obscure a Theory, and to supply the arduous Calculations relative to the next Transit in a most general Manner, I think cannot but be acceptable to every one who is a well Wisher to the Prosperity and Honour of his Country, as these are wholly founded in the Improvement of the Sciences in general, but most immediately in the practical Application of* Astronomy, Geography, *and* Navigation.

*I have also endeavoured to facilitate the difficult Ideas relative to the Phænomena of a Transit, by shewing how*

*they naturally ariſe from, and are ſolvable by the Properties of the Globe, and particularly the terreſtrial one; as alſo by a new Conſtruction of that Globe, by which all the Phaſes of a Tranſit, for any given Minute of Time, are inſtantly exhibited for all the Inhabitants of the Earth; for which Reaſon it juſtly merits the Title of a* TRANSIT GLOBE.

*Beſides three large Copper Plates of Figures and Diagrams, as a thorough Illuſtration of the general Subject, I have added a Map of the* Great South Sea, *to exhibit a View of the Iſland in that Part of the World where the Tranſit may be obſerved to the greateſt Advantage; the Author of it is Mr.* Pingrè, *who has given a large and particular Account of the Diſcovery and Condition of thoſe Iſlands, to which I refer ſuch Readers as underſtand* French. *But thoſe who have a Globe (eſpecially the* Tranſit Globe) *can have no Occaſion for any other* geographical Aſſiſtance, *to render the practical Part of this ſublime Study very eaſy and entertaining.*

*A* LECTURE *upon this Subject, which I propoſe to give, with Experiments on the above-mentioned Inſtruments, and other* new PIECES *of* MECHANISM, *is all that remains, as I apprehend, to render the* Doctrine of Tranſits *as univerſally intelligible, as it can ever be expected.*

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# CONTENTS.



CHAP.

## CHAP. V.



## CHAP. VI.



## CHAP. VII.



## CHAP. VIII.



## CHAP. IX.



CHAP.

## CHAP. X.



## CHAP. XI.



## CHAP. XII.



## CHAP. XIII.

# INSTITUTIONS OF ASTRONOMICAL CALCULATIONS.

## PART II.

## CHAP. I.

### *The* Astronomical Elements *of a* Transit *explain'd, with the* Rules *for predicting such Events.*

4002. THE first Element in the Astronomy of a Tranſit, is the *Motion of the Node* of the Planet's Orbit; Sir Isaac Newton, and other great Names, have declared poſitively that the Nodes of the Planet's Orbits are abſolutely at reſt, and Dr. Hadley ſpoke dubiouſly of it in his Diſſertation on the laſt Tranſit of Venus; but that Tranſit compared with the preceding one, obſerved by the famous Horrox in the Year 1639, put the Motion of Venus's Nodes paſt all

all doubt. And it has been ſince ſtated in Dr. HALLEY's Tables at 31″ *per Ann.* and by Mr. CASSINI it is made 34″, who ſuppoſes (from a Compariſon of a great Number of Obſervations) that Quantity muſt be very near the Truth.

4003. The Motion of the Nodes being Retrograde, and at the Rate of 34″ *per Ann.* the Place of it will be eaſily aſcertained by proper Tables for any given Moment of Time. Since in the laſt Tranſit (1761) the Place of the ☊ was 8ˢ 14° 40′, it will in 8 Years have retrograded 4′ 32″, therefore in the next Tranſit, 1769, the Place of the Node will be 8ˢ 14′ 35′ 20″.

4004. The Reaſon why the Venereal Tranſits ſucceed each other at ſuch different Intervals of Time, will be beſt underſtood by firſt defining the *Tranſit-Limits* for this Planet; in order to which let A B be part of the Ecliptic; D E part of Venus's Orbit interſecting the Ecliptic in the Node ☊; making the Angle A C D (as viewed from the Earth) = 8° 29′ 10″. Then the Semidiameter of the Sun is at this Node 15′ 45″, and that of Venus (as was found in the laſt Tranſit) is 29″, the Sum of both is 16′ 14″ = A D; hence in the Right-angled Triangle A C D, the Side A C is found = 110′ = 1° 50′, the double of which is 3° 40′ = A B, the whole Limit of the Tranſit. (See Fig. 1 and 2.)

4005. The next Thing to be conſidered is, that a Tranſit can only happen at *a Conjunction of the Sun and Venus at her interior Node*; therefore the Time of ſuch a Conjunction, or the Interval between two ſucceſſive ones, muſt be determined. Then ſince the Period of the Planet Venus is 224ᵈ. 16ʰ. 42′, and that of the Earth 365ᵈ. 5ʰ. 49′; it is evident, that if Venus and the Earth ſet out from any Right Line joining their Centers and that of the Sun, the Time that will lapſe before they arrive again to ſuch a Right Line will be equal to a Period of the Planet, and ſo much more as is equal to the Motion of the Earth in the Interval of two ſuch Conjunctions.

4006.

4006. Let the Earth's Motion in that Time be called A, then the Periodical Time of Venus is 5392,75 Hours, and that of the Earth is 8765,75 Hours; and we have this Analogy, *viz.* As $8765{,}75^{h.} : 5392{,}75^{h.} :: 360° + A : A$, and by Diviſion of Proportion, $3373 : 5392{,}75 :: 360 : A$. Whence we find $A = 575{,}57° =$ the Motion of the Earth between two Conjunctions; from which ſubduct 360°, there will remain $215°{,}57 = 7^{s}\ 5°\ 34'$, which is the Diſtance in the Ecliptic between every two Conjunctions of the Earth and Venus.

4007. Now 'tis evident, when ſuch a Conjunction happens at either Node, the ſame will happen again at the ſame Node when the ſaid Difference 215°,57 is ſo often repeated, or multiplied by ſuch a Number, that the Product ſhall exceed or fall ſhort of a Multiple of 360 by a Quantity leſs than the abovementioned Tranſit Limit. This will be beſt explained by an Example. The Tranſit in the Year 1761 was occaſioned by a Conjunction of Venus and the Sun near the deſcending Node, which was then in ☊ 14° 40′, and the Conjunction was in ☊ 15° 46′. If you multiply the Number 215,57 by 5, the Product is 1077,85, which is leſs than 1080 (= 360 × 3) by 2,15 = 2° 9′, which taken from ☊ 15° 46′, gives the Place of the Conjunction at the next Tranſit in ♐ 13° 37′, which is 47′ within the Limit B E above the Node ☋, as the laſt was within 44′ of the other Limit A D below it.

4008. Therefore it is evident another Tranſit muſt be produced at the ſame Node at the End of the Interval of 5 Conjunctions, or 8 Years nearly, and therefore muſt happen again in 1769, *June* 3. And this was alſo the Caſe in the Time of HORROX, when the Tranſit of Venus in 1631 was follow'd by another 8 Years after in 1639, which was the firſt ever obſerved, and that only by himſelf and his Friend Mr. CRABTREE.

4009. In like Manner, if the Number 215°,57 be multiplied by ſuch a Number as will give a Product that differs from a Multiple of 360°, by 180°, then will the Conjunction of the Earth and Venus fall within the Tranſit Limits of the oppoſite Node. Such a Number of Conjunctions is 81; for 215,57 × 81 = 17461,17, which exceeds 17280 (= 360 × 48) by 181; and therefore the 81ſt Conjunction falls but 1 Degree from the oppoſite Node.

4010. Now 81 Conjunctions take up the Space of 130 Years (for 5 : 8 : : 81 : 130 nearly) and conſequently the Tranſits of 1631, and 1639, produce at the Interval of 130 Years, the Tranſits of 1761, and 1769, at the deſcending Node ☋.

4011. The next Return of theſe Tranſits will be in a leſs Period of Time, *viz.* 113 Years; ſo that the Tranſit in 1761 will again happen in 1874, and the Tranſit of 1769 will again return in the Year 1882, both of them at the oppoſite or *aſcending Node*, and after 71 Conjunctions of the Earth and Venus.

4012. I ſhall here exhibit in one View all the Tranſits of Venus from the Year 1631 to 2368 incluſive, with the Interval of Years between each two, that the Regularity and Order in which they return, may be the better obſerved.

| Anno Dom. | Node. | Month. | Int. of Years. |
|---|---|---|---|
| 1631 | ☊ | Decemb. | |
| 1639 | ☊ | Decemb. | 8 |
| 1761 | ☋ | June | 122 |
| 1769 | ☋ | June | 8 |
| 1874 | ☊ | Decemb. | 105 |
| 1882 | ☊ | Decemb. | 8 |
| 2004 | ☋ | June | 122 |
| 2012 | ☋ | June | 8 |
| 2117 | ☊ | Decemb. | 105 |
| 2125 | ☊ | Decemb. | 8 |
| 2247 | ☋ | June | 122 |
| 2255 | ☋ | June | 8 |
| 2360 | ☊ | Decemb. | 105 |

2013.

4013. That it may further appear how accurately these Periods of 5, 71, and 81 Conjunctions bring about the Transits, even to the same Day of the Month for many Ages together, I shall here subjoin a Computation of that which will happen in 2368, the last in the Table, and distant from the present Time 600 Years.

| 4014. | | Sun's Place. | | Apogee. |
|---|---|---|---|---|
| An. Dom. 1768 | — | 6 11 42 48 | — | 3 8 46 55 |
| Years 600 | — | 11 24 26 32 | — | 10 6 40 |
| Novemb. 26 | — | 10 25 14 54 | — | 53 |
| Sun's Anomaly | — | 5 1 24 14 | — | 3 18 54 28 |
| Add Apog. | — | 3 18 54 28 | | |
| Sun's Place | — | 8 20 18 42 | | |

| 4015. | | Venus's Place. | | Node. |
|---|---|---|---|---|
| An. Dom. 1768 | — | 5 6 36 42 | — | 2 14 33 1 |
| Years 600 | — | 3 25 11 12 | — | 5 10 0 |
| Nov. 26 | — | 5 18 42 55 | — | 24 |
| Place of ♀ | — | 2 20 30 49 | — | 2 19 43 25 |
| Place of ☊ | — | 2 19 43 25 | — | 8 20 18 42 |
| | | 0 0 47 14 | — | 6 0 35 17 |

Thus it appears that Venus will be then within 47′ of the Node ☊, and therefore far enough within the Transit-limit. Also the Sun (or Earth) is but 35′ from the ☊, consequently the *Transit* will then be a very considerable one.

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## CHAP. II.

*The* ASTRONOMICAL CONSTRUCTION *of a* TRANSIT *upon the* SURFACE *of a* Celestial GLOBE. *The* NATURE *of* PARALLAXES *explain'd, with the general* METHOD *of* CALCULATION *for the* CENTER *of the* EARTH.

4016. THAT the Use of a *Transit of Venus* may the more fully appear, I shall give the *Astronomical Construction* of the

the next Tranſit as it may be repreſented upon the Surface of a *Celeſtial Globe* by the following Diagram of Circles, for the Beginning or Ingreſs in the Morning, and viewed on the North Side the Orbit of Venus. Let H Z O N be the general Meridian, H O the Horizon, and Z the Zenith of the Obſerver; P the Pole of the Equinoctial Æ L; Q the Pole of the Ecliptic T S K; R the Pole of the Orbit of Venus C S V; S the Place of the Sun and Venus near the Node; Z S M a Vertical paſſing thro' the the Sun; P S a Meridian thro' the Sun; Z P S the Hour Angle from Noon; Q S a Secondary of the Ecliptic to the Sun; R S a Secondary of the Orbit of Venus; Q P I the Solſtitial Colure. (Fig. 3.)

4017. Suppoſe the Time 9 o'Clock in the Morning, then in the Spherical Triangle Z S P there are known the Co-latitude Z P, the Angle Z P S, and the Sun's Co-declination P S, by which you find the Angle Z S P.

4018. Then in the Triangle Q S P there are known all the Sides, *viz.* P Q = 20° 30′, Q S a Quadrant, and P S the Co-declination of the Sun; alſo the Angle P Q S is known, as being the Sun's Diſtance S I from the firſt Point of *Cancer* I. From theſe Data the Angle P S Q is found, and thence the Angle Q S Z; alſo the Angle R S Q is known, being equal to that which the Path of Venus makes with the Ecliptic as ſeen from the Earth; and therefore the Angle R S Z is known, and of Courſe the Angle R S P.

4019. Therefore all the Parts are known neceſſary for a general Conſtruction of a Tranſit; but as the whole Doctrine and Uſe of the Tranſit depends upon a proper Reſolution of the Planet's *Parallax of Altitude* into the *Parallaxes of Longitude and Latitude*, and learning from thence the particular Quantities of Time to be added to, or ſubtracted from the Times in which the Phœnomena of the Tranſit happen as viewed from the Center of the Earth, in order to have the Times of the Ingreſs, Egreſs, Duration,

tion, &c. at the Places of Obſervation; it will be neceſſary next to give a Definition of thoſe Parallaxes.

4020. Let E S be a Right Line joining the Centers of the Earth and Sun in the Horizon, and P any Object in that Line as a Planet, &c. A E is the Semidiameter of the Earth. The Planet P will appear to an Eye at the Center of the Earth E upon the Surface of the Sun in F, but if viewed from the Surface of the Earth at A, it will be ſeen upon the Sun at G, and the Angle A P E is the *Horizontal Parallax* of that Planet. (See Fig. 4.)

4021. Suppoſe this Planet to be Venus in any other Poſition V in its Orbit above the Horizon E S, then its Place upon the Sun is at B when ſeen from the Center E, but viewed from the Surface of the Earth at A, it will appear upon the Sun at C; and the Angle A V E is the *Parallax of Venus* belonging to the Altitude P V, or Angle V E P; and *this Parallax* A V E *is to the Horizontal Parallax* A P E, *as the Co-ſine of the Altitude to the Radius.* (1899.)

4022. The Parallatic Angle at P or V is evidently that by which the Semidiameter A E of the Earth is ſubtended, or under which it appears at the Planet. Join A B, and the Angle A B E is that under which the Earth's Semidiameter appears from the Sun, and is therefore the *Sun's Parallax*, that great and important Object of our preſent Enquiries and Reſearches.

4023. The Difference between the *Parallax of Venus* A V E, or B V C, and the *Solar Parallax* A B E is the Angle B A C, and this is called the *Parallax of Venus from the Sun.* In the Triangle A V B, the Proportion of Diſtance between the Sun A B, and the Planet A V, being given, and the Sun's Parallax A B E aſſumed, the Parallax of Venus A V E will be found of Courſe; becauſe A B E : A V E : : A V : A B; and from thence the Angle B A C is alſo known.

4024. In order to adapt theſe general Principles to particular Caſes and Phaſes of the Tranſit, let us ſuppoſe the Globe

Globe on which the above Conſtruction is made (Fig. 3.) to be 40 Feet in Diameter, then the Degrees in a great Circle will be nearly four Inches long each, and conſequently the Disk of the Sun will be two Inches Diameter, which therefore let be repreſented by the circular Area B E L F H; S the Center, in which the Lines P S, Q S, R S, meet, from the Pole P of the Equator, Q of the Ecliptic, and R of Venus's Orbit. Let L N be part of the Ecliptic near the Node N, and M N the Path of Venus in which ſhe is deſcending to the Node over the Face of the Sun in the Chord E B. (Plate I. Fig. 5.)

4025. Draw E S and B S; then the Right-angled Triangles N S D and C S D are ſimilar, the Angle at D being common to both; and the Angle at N = C S D which is known; alſo the Side D S is known, being the *geocentric Latitude* of Venus at the Moment of the Conjunction; and from thence C S and C D will be found.

4026. The Chord or Tranſit-line E B is biſected by the Perpendicular R S, and the Lines S B and C S are known, and therefore the Side C B (= C E;) and likewiſe all the Angles in the Triangles C S B, C S E, will become known.

4027. Again, let S V be the Sum, and S G the *Difference* of the *Semidiameters* of the Sun and Planet; theſe are both known for the Time of the Tranſit; therefore in the Triangles C S V, C S G, the Sides C V, C G, will be known with all the Angles in each.

4028. Hence alſo the Lines G V, A I, will be known both in Motion and Time; and alſo their Parts A B, B I; and G E, E V. Thus the Lines D V, D E, D G, and D I, D B, D A, are known; and conſequently the Times of the External Contact at V, and the internal Contact at G, will be known at the *Ingreſs*; alſo the Time of the Middle of the Tranſit at C; and at the internal and external Contacts at the Egreſs at I and A; and of the whole

Duration of the Tranſit from V to A, as ſeen from the *Center of the Earth.*

## CHAP. III.

*The general* METHOD *of calculating the* PHASES *and* DURATION *of a* TRANSIT *with regard to any particular* PLACE *upon the* EARTH'S SURFACE, *by Means of the* PARALLAXES *in* LONGITUDE *and* LATITUDE.

4029 TO calculate the Phænomena of the Tranſit as they appear to an Obſerver at Z upon the Earth's Surface, it is neceſſary next to aſcertain the Quantity of all the Lines P A, P B, P I, P D, P G, P E, P V; and of the adjacent Angles P V N, P B N, P G N, P D N, &c. which it is eaſy to do, thus; in the Triangle P D S, the two Sides S D and S P, with the included Angle D S P, are known, which therefore give the Side D P, and the Angle S D P; whoſe Complement P D Q added to the known Angle S D N, or Q D M, will give the Angles P D N, and P D M. Then in the Triangle P D G, the Sides P D, D G, and the Angle P D G, are known, which give the other Side P G, and the Angle at G. Thus in all the other Triangles P D E, P D V, P D I, P D B, P D A, there being two Sides and an Angle given, the remaining Side and Angles will be known.

4030. Let P V, R V, Z V, be drawn from the Poles P R Z, (of the Equator, Orbit, and Horizon) to the Planet V; then in the Triangle Z V P, the Side Z P = Co-Latitude; the Side P V has been already found (4029); and the Angle Z P V is the Time from Noon of the Beginning of the Tranſit, all which are therefore known; hence we find the Zenith Diſtance Z V of Venus, and the Angle of Poſition Z V P.

4031. Having the Zenith Diſtance Z V of the Planet, the Parallax belonging to that Altitude is known by the Analogy in (4021) let this Parallax be denoted by V X in in the Line Z V continued out, therefore when the Planet is ſeen from the Center of the Earth at V, it will by this Parallax be depreſſed to X, as viewed by an Obſerver at Z. Thro' the Point X, and parallel to the *true Path* (or Tranſit-Line) M N, draw the dotted Line *m n*, which will be the *apparent Path* of Venus, or that in which ſhe appears to move, as ſeen from the Earth's Surface at Z. (See Plate II. Fig. 1.)

4032. This Parallax V X is the Meaſure of the Angle V A B in Plate I. Fig. 4. and conſequently if the true Quantity thereof could be eſtimated by any Aſtronomical Inſtrument, or Methods hitherto uſed for that Purpoſe, the Parallax of the Sun A B E would be found very near the Truth; for in the Triangle A V B, we have A B : V B : : V A B : A B V, or A B E.

4033. But no Means hitherto uſed to determine the Parallax of any Planet have proved ſucceſsful; and Aſtronomy can furniſh but one other Method more, by which the *Solar Parallax* can be attempted with any Probability of approaching to it near the Truth; and this is by a *Tranſit of Venus*; for the Tranſit of *Mercury* does not afford a *Parallax of that Planet from the Sun* ſufficient to anſwer any ſuch Purpoſe.

4034. In this Tranſit, the *Parallax of Altitude* V X is eſtimated by the Effect which it produces in *the different Times of the Phaſes*; for this Parallax when properly reſolved, will ſhew a Difference in the Time when any *Ingreſs*, *Egreſs*, &c. will happen to an Eye placed in the Center of the Earth, and any where upon its Surface at Z, both in regard to the *Latitude* and *Longitude* of that Place.

4035. Thus (by the Principles of *Mechanics*) the Parallax V X is reſolvable into the two others, X *v*, and V *v*; the

the firſt is a *Parallax in Longitude*, becauſe to a Perſon at Z the Planet will appear at X, which is diſtant from the Secondary R V, to the *Eaſt* or *Weſt*, by the Quantity of the Line X *v*; and therefore the Moments of Time in which the Planet will be ſeen in the two Secondaries paſſing through V and X, will be different according to the *Longitude* of the Spectator.

4036. The other Part V *v* is the *Parallax of Latitude*; for while the Planet is ſeen at V from the Earth's Center, it is depreſſed in the Secondary R *v* to the Point *v* in the apparent Path. But it is evident, if the Planet be in Contact at V, it cannot be ſo in any other Point *v* in the ſame Circle; but the Moment of Contact will be ſooner or later than at V, by the Difference of Time the Planet takes to paſs from *v* in the Secondary to the Point Y in the Circle of Contact Y D V, joining the Centers when in Contact at V and Y in the *true and apparent Paths*.

4037. The Times which the Planet takes to paſs thro' the Spaces X *v* and *v* Y, will be known from a Computation of the Quantity of the ſaid Lines, which is thus. The Angle Z V P is known (4030) alſo the Angle R V P, $= 90° -$ P V M, therefore alſo, the Angle Z V R $=$ X V *v*; hence in the Right-angled Triangle *v* V X, having the Hypothenuſe V X, and the Angle at V, the Sides X *v*, and V *v* become known.

4038. Then for computing the Value of *v* Y, it is evident, by drawing S Y, we have Y *v* $=$ *c* Y $-$ *c v* $=$ *c* Y $-$ C V. But C V is known, being half the Motion of the Central Tranſit; and *c* Y is $= \sqrt{S\,Y^2 - S\,c^2}$ a known Quantity; for S *c* $=$ S C $-$ C *c* $=$ S C $-$ V *v*, both which are known. Alſo S Y is known being the Sum (or Difference) of the Semediameters of the Sun and Venus; and hence *v* Y is known.

4039. The Motions *v* X and *v* Y being known, the Times in which they are performed are known, from the known geocentric Motion of Venus at the Time of the

Tranſit. In all that has been hitherto ſaid, the *Longitude* of the Obſerver, or of the Point Z, is its Diſtance *Eaſt* or *Weſt*, from the Secondary R V; and the *Latitude* thereof is its Diſtance, *North* or *South*, from the Orbit of Venus.

4040. Therefore the *Motion in Longitude* ariſing from the *Parallax of Longitude* is to be reckoned from or towards the Secondary R V, in the true Path (or Orbit) M N; but the Motion in Latitude, reſulting from the *Parallax of Latitude*, is conſidered in the apparent Path *m n*. We ſhall now exhibit, in four Diagrams, the *eight* different Phaſes of the INGRESS and EGRESS of the Planet upon the Diſk of the Sun in the next Tranſit, which will be found, if duly conſidered, to comprehend all the Varieties incident to that wonderful Phænomenon; the Conſtruction being the ſame for the interior Contact at every *Ingreſs* or *Egreſs*, requires not to be drawn, as it would too much embarraſs the Diagrams with unneceſſary Lines.

4041. When the INGRESS is viewed by an Obſerver at Z, on the *North Side* of the Planet's Orbit, in the *Eaſtern Hemiſphere*, or *Forenoon*, then the Conſtruction of it will be ſuch as we have deſcribed (in Fig. 1. Plate II.) Wherein it is evident, that *v* X is the Motion in Longitude, and *v* Y, that in Latitude; and the Times of paſſing theſe Lines or Spaces, are thoſe by which the true Time of the Ingreſs at V will be varied, by the Addition or Subtraction of their Sum or Difference, to or from the ſaid Time of Ingreſs viewed from the Center of the Earth: Whence will ariſe the following Caſes.

4042. *Firſt*; When the Obſerver is ſituated upon the Orbit of Venus (or rather the Parallel ſhe deſcribes) he has then no Latitude; the Point *v* coincides with V; and the *Parallax of Altitude* is wholly a *Parallax of Longitude* in the true Path M N.

4043. *Secondly*; If the Obſerver's Situation be in the Planet's prime Vertical R V, he has then no Longitude, and of Courſe there will be no *Parallax of Longitude*, but

of

of *Latitude* only, *viz.* V *v*; the Line or Azimuth Z X, coinciding with R *v*, makes *v* X = 0; and *v* Y is the only Effect of Parallax in this Case.

4044. *Thirdly*; A Situation any where between the Orbit and its Secondary R V, will give a Parallax of Altitude V X in the Azimuth Z V, productive of a Parallax of *Eaſtern Longitude* *v* X, and a Parallax of Latitude V *v*.

4045. *Fourthly*; When the Spectator is between the Secondary R V, and the Circle of Declination P V, the the Parallax of Longitude will be on the *Weſt Side* of the Circle R V. Theſe are all the Caſes which require to be ſpecified for the Ingreſs, at preſent, in regard to the Poſition of the Obſerver.

4046. The next Thing is to conſider how the *Time of Ingreſs* is affected by theſe different and variable Parallaxes. The true Time of Ingreſs is in the Secondary R V, when the Planet is at V; the Time of apparent Ingreſs is the Moment of the Planet's being at Y in the apparent Path. Hence alſo will ariſe four Caſes in regard to the Time.

4047. *Firſt*; When the Point X falls *Eaſtward* of the Circle R *v*, it is evident the Time of the Ingreſs will be *retarded* by all the Time required in paſſing through the Space X *v*, or Parallax of Longitude.

4048. *Secondly*; When the Point X falls to the Weſt of the Circle R *v*, the *Time of Ingreſs* will be accelerated by the *Parallax of Longitude*.

4049. *Thirdly*; When the Point Y (in which the Center of the Planet is at the Moment of apparent Ingreſs) falls *Eaſtward* of R *v*, the *Time of Ingreſs* will be *accelerated* by the *Parallax of Latitude* *v* Y.

4050. *Fourthly*; But when the Point Y is on the *Weſt* Side of the Circle R *v*, then the Time of Ingreſs will be *retarded* by the Time the Planet takes in paſſing from *v* to Y.

All theſe Caſes are general, and hold good for all Ingreſſes,

greſſes, Egreſſes, interior Contacts, *&c.* whereſoever obſerved.

4051. From what has been premiſed, it eaſily appears (1) That when X and Y fall on different Sides of R *v* the Time of Ingreſs, Egreſs, *&c.* will be *accelerated* or *retarded* by the Sum of both the Parallaxes, or by the Time of paſſing over X *v* + *v* Y, = X Y. (2.) That when the Points X, Y, fall both on the ſame Side the Circle R *v*, then the *Time of Ingreſs*, &c. will be *accelerated* or *retarded* by the Difference of the Parallaxes, or by the Time of paſſing over X *v* — Y *v* = X Y.

4052. And ſince only the ſame Lines Z X, R *v*, and P V; and the ſame Points X, *v*, Y, are concerned in the Ingreſs, Egreſs, *&c.* of the Planet, as viewed in the *Eaſtern* or *Weſtern Hemiſphere*, or on the *North* or *South Side of the Orbit*; therefore it will be evident, by Inſpection only, when the Time reſulting from the Parallaxes is *to be added to*, or *ſubtracted from the true Time*, in order to have the *Moment of viſible Ingreſs*, or *Egreſs*, or *Quantity of apparent Duration*. And to make every Thing in this intricate Doctrine clear and eaſy, we ſhall next illuſtrate and apply the foregoing Theory of a Tranſit, by a Calculation of that which is next to happen, for the Time of its *Ingreſs*, as ſeen from the Center of the Earth.

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## CHAP. IV.

*The foregoing* METHOD *of* CALCULATION *exemplified and illuſtrated, in a* COMPUTATION *of the* PHÆNOMENA *of the enſuing* TRANSIT *of* June 3, 1769, *in reſpect to the* CENTER *of the* EARTH.

4053. IN Order to this, the Places of the Sun and Venus muſt be found for ſome particular Point of Time, near the Middle of the Tranſit, but it may be either before or after.

We

We ſhall here chuſe the Moment of *Eight o'Clock*. From MAYER's Solar Tables (in the preceding Part of theſe *Inſtitutts*) the Place of the Sun is had as below.

| | ☉ Long. | | | | Apogee. | | | |
|---|---|---|---|---|---|---|---|---|
| | ♃ | ° | ′ | ″ | ♃ | ° | ′ | ″ |
| An. 1769 — | 9 | 10 | 24 | 12 | 3 | 8 | 56 | 52 |
| June 3 — | 5 | 1 | 47 | 23 | | | | 27 |
| Hour 8 — | | | 19 | 43 | | | | |
| | 2 | 12 | 31 | 18 | 3 | 8 | 57 | 19 |
| | 2 | 12 | 31 | 18 | 2 | 12 | 31 | 18 |
| Equat. add — | | | 50 | 7 | | | | |
| | | | | | 11 | 3 | 33 | 59 |
| The Sun's true Place | 2 | 13 | 21 | 25 | | | | |

4054. From Dr. HALLEY's Tables of Venus's Motions, her Place for the ſame Time is had as follows.

| | Long of ♀ | | | | Aphel. | | | | Node ☊ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1769 — | 0 | 23 | 0 | 19 | — 10 | 7 | 36 | 26 | — 2 | 14 | 33 | 32 |
| May 23 — | 7 | 19 | 6 | 36 | | | | 17 | | | | 10 |
| Hour 8 — | | | 32 | 3 | | | | | | | | |
| | 8 | 12 | 38 | 58 | 10 | 7 | 37 | 13 | 2 | 14 | 33 | 42 |
| | 8 | 12 | 38 | 58 | 8 | 12 | 38 | 58 | 6 | | | |
| Equat. add | | | 39 | 6 | | | | | | | | |
| | 8 | 13 | 18 | 4 | 10 | 5 | 1 | 45 | 8 | 14 | 33 | 42 |
| | | | | | Mean Anomaly. | | | | Place of ☊ | | | |
| Reduct. add — | | | | 7 | | | | | | | | |
| Place in the Ecliptic. | 8 | 13 | 18 | 11 | | | | | | | | |

| | ♃ | ° | ′ | ″ |
|---|---|---|---|---|
| 4055. The Place of the Earth is - - - | 8 | 13 | 21 | 25 |
| From which ſubduct the Place of ♀ | 8 | 13 | 18 | 11 |
| There remains the Difference - - | | | 3 | 14 |

4056. The hourly Motion of Venus in the Ecliptic will (being ſo near the Node) be the ſame with that in her Orbit, *viz.* 4′. The hourly Motion of the Earth at the Time of the Tranſit is 2′ 23″,5 as will appear from any Ephemeris.

If then we ſay, as the Difference of the hourly Motions 1′ 36″,5 = 96″,5 is to the Difference of their Places 3′ 14″ = 194″ ſo is the hourly Motion of Venus 4′ = 240″ to her Diſtance from the Plane of Conjunction 482″,5 = 8′ 2″,5. This being added to Venus's Place, gives the Point 8ˢ 13° 26′ 13″ ½ wherein the Conjunction happens.

4057. The Time in which Venus paſſes over the Space of 482″,5 is thus found: As 240″ : 60′ : : 482′,5 : 120′,63 = 2° 0′ 38″; this added to the given Time of 8 o'Clock, gives 10° 0′ 38″ for the mean Time of Conjunction, at the Center of the Earth.

| | ˢ | ° | ′ | ″ |
|---|---|---|---|---|
| 4058. From the Place of the Node ☊ = | 8 | 14 | 33 | 42 |
| Subduct the Place of Conjunction | 8 | 13 | 26 | 13,5 |
| There remains the Diſtance from ☊ = | 1 | 7 | 28,5 | |

4059. The Inclination of Venus's Orbit to the Plane of the Ecliptic, being 3° 23′ 20″, her heliocentric Latitude at the Conjunction will be = 4′.

4060. The mean Diſtance of the Earth from the Sun being put = 10000, it's Diſtance at the Time of the Tranſit will be 10151; the Diſtance of Venus from the Sun will be 7262,6; and her Diſtance from the Earth 2888,5. Therefore ſay: As 2888,5 : : 7262,6 : : 4′ : 10′ 3″, = Venus's geocentric Latitude at the Conjunction; and is the Meaſure of the Line S D in the Diagrams of the Tranſit.

4061. The Inclination of Venus's Orbit to the Ecliptic will likewiſe be magnified in a *geocentric View*, in the inverſe Ratio of the Diſtances of the Sun and Earth; therefore ſay: As 2888,5 : 7262,6 : : Tangent of 3° 23′ 20″ : Tangent of 8° 28′ 6″ = M N L, which is alſo the Quantity of the Angle R S Q. (Fig. 5. Plate I.)

4062. The Obliquity of the Ecliptic being 23° 29′, and the Sun's Place therein 73° 26′ 13″ found as above (4053.) You from thence find the Sun's Declination 23° 27′ 15″ ½; and its Co-declination is therefore S P = 67° 32′ 44″ ½.

4063.

4063. In the Spherical Triangle P S Q all the Sides are known; and from thence you find the Angle P S Q $= 7^\circ\ 11'\ 37''\frac{1}{2}$; This added to the Angle R S Q, gives the Angle R S P $= 15^\circ\ 39'\ 43''\frac{1}{2}$.

4064. In the Triangle S D C, there are known the Side S D $= 10'\ 3''\frac{1}{2}$, the Angle D S C $= 7^\circ\ 11'\ 37''\frac{1}{2}$, and the Right Angle at C; from which you find the Side C D $= 1'\ 29''$, and the Side C S $= 9'\ 57''$. Alſo the Angle C D S $= 81^\circ\ 31'\ 54'' =$ Q D M.

4065. In the Right-angled Triangle S C V, there are known the two Sides S C $= 9'\ 57''$, and S V $= 16'\ 14''$ $= 15'\ 45'' + 29''$, Sum of the Semidiameters of the Sun and Venus; and from thence we have the Side C V $= 12'\ 49''$; and therefore C V $-$ C D $=$ D V $= 11'\ 20''\frac{1}{2}$. Alſo the Angle C V S $= 37^\circ\ 47'\ 46''$, and the Angle C S V $= 52^\circ\ 12'\ 14''$.

4066. In the Triangle C S G we have given the Side C S, and the Side S G $= (15'\ 45'' - 29'' =)\ 15'\ 16''$ $=$ the Difference of the Semidiameters of the Sun and Venus; from which we find the Side C G $= 11'\ 34''\frac{1}{2}$; whence alſo D G $= 10'\ 5''\frac{1}{2}$.

4067. The heliocentric Motion of Venus from the Earth is (as before obſerved) $1'\ 36''\frac{1}{2} = 96'',5$; and her geocentric Motion from the Sun will exceed that in the Ratio of 7262,6 to 2888,5; hence her apparent Motion upon the Sun's Diſk will be $4'\ 2''\frac{1}{2}$ per Hour.

4068. Therefore $242''\frac{1}{2} : 60' :: 769''\frac{1}{2} : 3^h\ 10'\ 19''$, the Time of Venus paſſing through C V, which therefore is the *Half-duration of the Tranſit*, and ſo the *Whole Duration* would be $6^h\ 20'\ 38''$, if ſeen from the Center of the Earth.

4069. The Time of her paſſing through D V will in like Manner be found $= 2^h\ 48'\ 19''$, which taken from the Time of the Conjunction $10^h\ 0'\ 38''$, leaves $7^h\ 12'\ 19''$ for the *Time of the Beginning*, or *external Contact* at V.

4070. The Time of paſſing through G D is found by

 the

the ſame Analogy to be $2^h\ 29'\ 43''$, which ſubducted from the Time of paſſing through D V will leave $18'\ 36''$ for the Time between the *external Contact* at V, and the *internal Contact* at G, which therefore will happen at $7^h\ 30'\ 55''$.

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## CHAP. V.

*The* METHOD *of calculating any* PHASE *of a* TRANSIT *for a particular* PLACE, *exemplified in a* COMPUTATION *of the* TIME *of apparent* INGRESS *in the* TRANSIT *of* June 3, 1769, *as ſeen at the* CITY *of* LONDON.

4071. WE next proceed to the Calculation of the Phænomena of the Tranſit for any particular Place upon the Surface of the Earth, thereby to find what Differences will ariſe from the Effects of *Refraction* in regard to the Times of their Appearance and Duration. This I ſhall exemplify in a Computation of the BEGINNING of the Tranſit, for the exterior and interior Contact, as they will be obſerved by a Spectator at the City of LONDON, denoted by Z. (Fig. 5. Plate I.)

4072. Then in the Triangle D P S there are known the two Sides S P and S D, and the included Angle at S; by which we find the other two Angles S D P $= 172°\ 47'\ 52''\ \frac{1}{2}$ and D P S $= 1'\ 22''\ \frac{1}{2}$; and alſo the Side D P $= 67°\ 23'\ 8''$. The Complement of S D P to $180°$ is the Angle P D Q $= 7°\ 12'\ 7''\ \frac{1}{2}$, which being taken from Q D M (= C D S $= 81°\ 31'\ 54''\ \frac{1}{2}$) there will remain the Angle P D M $= 74°\ 19'\ 47''$.

4073. Hence in the Triangle P D G, there are given the two Sides D P, D G, and the included Angle D; by which we find the Side P G $= 67°\ 20'\ 25''$. Alſo in the Triangle P D V, the Sides P D and D V are known, and the ſame Angle at D; therefore we find the Side P V $= 67°\ 20'\ 4''$. Again, in the ſame Triangles, are found the

the external Angles P G M = 74° 23′ 5″, and P V M = 74° 23′ 34″.

4074. In the Triangle Z P V are known the Side Z P = 38° 29′; the Side P V = 67° 20′ 4″; and the included Angle Z P V = 108° 4′ 45″ = 7ʰ 12′ 19″ the Time from Noon (4069) of the beginning of the Tranſit. From thence we find the Side Z V = 83° 12′ 48″, the Diſtance of Venus from the Zenith of LONDON. Her *Altitude* therefore at that Moment is 6° 47′ 12″.

4075. In like Manner the Moment of interior Contact, being 7ʰ 30′ 55″, gives the Angle Z P G = 112° 43′ 45″, *&c.* then with the Side Z P and P G, we find the third Side Z G = 85° 25′ 42″, the Zenith Diſtance of the Planet at G.

4076. In the ſame Triangles we find the Angle Z V P = 36° 4′, and the Angle Z G P = 35° 9′. If the Angle P V M be taken from the Right Angle R V M, there will remain the Angle R V P = 15° 36′ 26″, which added to the Angle Z V P, gives the Angle R V Z = 51° 40′ 26″. In the ſame Manner you find the Angle R G Z = 50° 45′ 55″. (See Fig. 2. Plate II.)

4077. If we now aſſume the *Sun's Parallax* A G E = 10″, we ſhall have the horizontal Parallax of Venus A P E = 35″,143; and from thence the Difference = 25″,143 = F E G, the horizontal Parallax of Venus from the Sun. (See Fig. 4. Plate I.)

4078. Hence the Parallax belonging to the Zenith-diſtance Z V will be found = 24″,966; and that for the Diſtance Z G = 25,″063; therefore 25″, which is the Mean, may be uſed for either of them indifferently without Error, and ſo we have V X = 25″. (See the Diagrams, Plate II.)

4079. Therefore in the Right-angled Triangle *v* V X, there are known the Side V X, and the Angle X V *v* = R V Z; by theſe are found the Side X *v* = 19″,61; which is the *Parallax in Longitude*; and the Side V *v*

$= 15'',503$, which is the Parallax in Latitude. And because $C c = V v$, therefore $S c = 9' 41'',57$. (4064.)

Therefore $\sqrt{SY^2 - Sc^2} = cY = 13' 1'',32$ from which take $CV = 12' 49'',5$, and there will remain $vY = 11''82$ (4038.)

4080. This Method of finding the Value of $vY$ is indeed ſtrictly mathematical; but the following is much eaſier, and very exact. The Arch $VY$ may be taken for Part of the Tangent to the Circle $DVL$ in the Point $V$; alſo the Angles $vVX + vVY = 90° = vVX + SVC$; therefore the Angle $vVY = SVC$; and is given by (4065.) Hence the Parallax of Latitude $Vv$ being known, its Effect $vY$ is known alſo by this Analogy. As the Radius to the Tangent of $vVY$, ſo is $Vv$ to $vY$.

4081. The hourly Motion of Venus from the Sun is $242'',5$ as we have ſhewn (4068) therefore ſhe will paſs over the Line or Space $vX = 19'',61$ in $4' 51'',12$ of Time, which will be the Effect of the *Parallax in Longitude*. Alſo the Time in paſſing over the Space $Yv$ will be $2' 55''44$; and this is the Effect of *Parallax in Latitude*. Therefore the whole Effect of Parallax is the whole Time of paſſing through $YX$, which is $7' 46'' \frac{1}{2}$; and by ſo much will the Time of the Ingreſs or external Contact happen ſooner to an Obſerver at *London* Z, than to an Eye placed in the Earth's Center; and therefore we may expect to ſee the Tranſit begin at $7^h 4' 32'', \frac{1}{2}$ in the *Evening*.

4082. In the ſame Manner it will be found, that the Acceleration of the Time of the *internal Contact* at G will be $4' 47''$ by the Parallax of Longitude, and $3' 20''$ by that of Latitude. So the whole Acceleration will be $8' 7''$, and therefore we may expect the interior Contact to happen at $7^h 22' 48''$. Hence the Time between the external and internal *apparent Contacts* will be $18' 16''$; ſhorter than the ſame Interval viewed from the Earth's Center by $20''$. (4068.)

CHAP.

## CHAP. VI.

### *The* METHOD *of computing the* Maxima *and* Minima, *or the greateſt and leaſt* EFFECTS *of* PARALLAX *reſpecting the* TIMES *of the* PHASES *and* DURATION *of the* TRANSIT *to* OBSERVERS *in different Parts of the* EARTH'S SURFACE.

4083. WE have ſhewn that there will be conſiderable Differences between the *true and apparent Times* of the Phaſes and Durations of the Tranſit, and how they may be eſtimated at any particular Place; but we are further to remark, that theſe Differences are *greater or leſs*, according to the different Circumſtances or Situation of the Obſerver with regard to *Latitude and Longitude*; the *Eaſtern or Weſtern Hemiſphere*, the *North or South Side of the Orbit*, &c. And it will appear that there is one Situation of the Obſerver which will make the Difference of Time between his obſerved Phaſe and the central one the greateſt poſſible, or a *Maximum*; and conſequently theſe Places are the principal Objects of our preſent Enquiry.

4084. This *Maximum*, 'tis evident, conſiſts in the largeſt Quantity of the Line Y X, which again depends upon a particular Quantity of the Angle X V *v*, or rather of the Angle X V Y; for ſince the Angle V Y X, and the Line X V are given Quantities, the Caſe is reduced to this *Problem*, viz. *to adapt a given Line* X V *in a given Angle* Y, *ſo as to make the Side* Y X *a Maximum*. To ſolve which, let the two equal Lines X V and A B have two different Poſitions in the given Angle V Y X (Fig. 6. Plate I.) then, by Trigonometry, we have A B : Sine of Y : : A Y : Sine B; and X V : Sine of Y : : X Y : Sine of V; therefore we have A Y : *s* B : : X Y : *s* V; from whence it appears that the Side A Y encreaſes with the Sine of the Angle B, and is a *Maximum* when that Sine is greateſt; that is,

when

when the Point B arrives at V, where the Angle X V Y is a right one, and the Side X V, of Courſe, a *Maximum*.

4085. Hence then, (in the Diagrams of Plate II.) it is evident, the Secondary Z V, and the Radius S V, muſt, in this Caſe, coincide, and make one right Line; or in other words, the *Secondary paſſes thro' the Centers of Venus and the Sun, when the whole Effect of Refraction is the greateſt poſſible*; therefore the Angle Z V R = *v* V X = V S C = 52° 12′ 14″, at the enſuing Tranſit.

4086. Again, the greater the Zenith Diſtance Z V is, the greater will be the Refraction V X in Altitude, and therefore Obſervations of any particular Phaſe made as near the Horizon as conveniently can be, will be ſo much the better; we ſhall therefore ſuppoſe that the leaſt Altitude that will admit of a diſtinct View of the Contact of the Limbs of Venus and the Sun, is 7 Degrees; and in that Caſe the Refraction X V = 25″, extremely near. (4021.)

4087. In the next Place, we are to determine from the above Principle, where thoſe two Places are, in one of which the Time of any Phaſe is *accelerated* as much as poſſible, and in the other, it is in the ſame Manner *retarded*; for in theſe Places there will be the greateſt Difference of Time in the Obſervations that can be obtained.

4088. By the Conſtruction of the Tranſit (Plate II. Fig. 2.) it appears that the Ingreſs will be *accelerated* by the Sum of both Parallaxes Y X in the *Afternoon* to an Obſerver on the *North Side of Venus's Orbit*. And in (Fig. 3.) it is evident, the ſame Phaſe will be *retarded* equally from the ſame Cauſe to an Obſerver on the *South Side* of the ſaid Orbit in the *Morning*. The Latitude and Longitude of each of theſe Places, is therefore to be determined by the following Proceſs.

4089. In the firſt Caſe, becauſe the Angle R V Z = 52° 12′ 14″ for a *Maximum*; therefore its Complement Z V M = 37° 47′ 46″ ſubducted from the Angle P V M = 74° 23′ 34″ will leave the Angle P V Z = 36° 35′ 48″.

Again,

Again, by Suppoſition, the Side Z V = 83°; and the Side P V = 67° 20′ 4″. Therefore in the Triangle P V Z, having given two Sides and the included Angle, we find the third Side Z P = 38° 31′ 43″ whoſe Complement is the *Latitude of the Place* Z = 51° 28′ 17″, which is the firſt Thing required.

4090. Then for the *Longitude of the Place* Z, we find in the ſame Triangle, the Angle Z P V = 108° 10′ 57″, which is the *Longitude of the Place* Z, with regard to the Meridian P V, which paſſes thro' Venus at the Beginning of the Tranſit. But ſince that Angle with reſpect to *London*, is 108° 4′ 45″; it is plain, the *Meridian of the Place* Z is but 6′ 12″ eaſtward of the *Meridian of London*. And therefore ſince the Latitude and Longitude of the Place Z, is ſo very little different from the Latitude and Longitude of LONDON; *this great City has happily the very beſt Situation that poſſibly can be to obſerve the moſt favourable Ingreſs that can, in any Tranſit, ever happen.*

4091. It remains now to find the Place where the ſame Phaſe or Ingreſs is *retarded as much as poſſible by parallatic Time*. And this (as is evident) from (Plate II. Fig. 3.) will be found on the *South Side of the Orbit*, and in the *Eaſtern Hemiſphere*, or *Morning Obſervation*.

4092. To compute the Quantity of this *maximum* Retardation, you proceed as follows; the Angle *v* V X = *r* V Z 52° 12′ 14″ as before; alſo the Angle *p* V N (= P V M) = 74° 23′ 34″, from which if you deduct Z V N = 37° 47′ 46″, there will remain the Angle *p* V Z = 36° 35′ 48″, as in the *London* Ingreſs.

4093. Then in the Triangle *p* V Z, the Side *p* V = 112° 39′ 56″ (being the Complement of P V in the other Caſe;) alſo the Side Z V = 83°, by Suppoſition. Therefore, by having theſe two Sides, and the included Angle at V, you find the third Side *p* Z = 46° 29′ 27″, whoſe Complement 43° 30′ 33″ is the *South Latitude* of the Place Z of Obſervation.

4094.

4094. The *Longitude of the ſaid Place* is known from the ſame Data; for thereby the Angle V *p* Z, will be found $= 54^{\circ}\ 44'\ 30''$, which ſhews the Poſition of the Meridian paſſing through Z is ſo much to the *Weſt* of that paſſing through the Planet V at the Moment of Ingreſs; but this through Venus is diſtant *Weſtward* from *London* $108^{\circ}\ 4'\ 45''$, as we have ſhewn (4075.) Therefore, the *Longitude ſought* of the Place Z from *London*, is $162^{\circ}\ 49'\ 15''$ *Weſtward.*

4095. The Quantity of the Angle Z *p* V $= 54^{\circ}\ 44'\ 30''$ turned into Time, is $3^{h}\ 38'\ 58''$, which is the Time from Noon, and therefore ſubtracted from XII, leaves $8^{h}\ 21'\ 2''$, for the Time of the Morning when the Central Ingreſs happens at that Place, *viz.* upon the *4th Day of June*, at $21'\ 2''$ *after* 8 *o'Clock.*

4096. Then for the Difference between the central and apparent Time, it will be nearly the ſame as at *London*, for in the Triangle X V *v*, the Side V X and the Angle at V are the ſame here as they were there; and conſequently the Sides X *v*, and V *v*, will be found the ſame, *viz.* X *v* $= 19'',61$; and V *v* $= 15,''5$; but becauſe S *c* is here $=$ S C + V *v*, we ſhall find *c* Y $= 12'\ 37''$, which taken from C V $= 12'\ 49''\ \frac{1}{2}$, leaves Y *v* $= 12''2$, not $\frac{2}{3}$ of a Second larger than before at the *London* Ingreſs. Therefore the Ingreſs will be retarded here by $7'\ 46''\ \frac{1}{2}$, and ſo the Time of obſerving it at the Place Z, will be $8^{h}\ 28'\ 48''\ \frac{1}{2}$ in the *Forenoon.*

4097. The Proceſs is the ſame in calculating the Times of a *retarded and accelerated Egreſs*; for it appears that the Egreſs will be *retarded* by the *joint Effect of both Parallaxes*, when viewed in the Morning on the *North Side* of the Orbit of Venus; and the ſame Egreſs will be in the ſame Manner *accelerated* when obſerved in the *Afternoon* on the South Side of the Orbit, as in Fig. 1. and 4. Plate II.

4098. To this End the ſame Lines and Angles muſt be calculated for the Place of Venus at A, in her external Contact with the Sun's Limb at her *going off*, as was done

at V for her *coming on*, and theſe will be had from the ſame Data. Thus we find A P = 67° 27′ 2″; the Angle P A M = 74° 13′ 15″; its Supplement P A N = 105° 46′ 45″, P A R = 15° 46′ 45″. The *Maximum* Angle Z A R = *v* A X = 52° 12′ 14″, and therefore the Angle Z A P = 67° 58′ 59″; alſo the Side Z A = 83°, as before.

4099. Then in the Triangle Z A P, there are given the two Sides A P and A Z, with the included Angle Z A P, by which we find the the third Side Z P = 67° 1′ 23″, whoſe Complement is 22° 58′ 37″, the *Latitude* North, for the Place of the Obſerver at Z.

4100. In the ſame Triangle we find the Angle A P Z = 88° 11′ 30″, which is the Longitude of the Meridian of the Obſerver *Weſtward* from that which paſſes through the Planet at A. This converted into Time, is 5ʰ 54′ 46″, which taken from XII, leaves 6ʰ 5′ 14″, for the Time in the Morning of *June* the 4th, when the *Tranſit ends there*, as ſeen from the Center of the Earth.

4101. And becauſe the Angle *v* A X, and the Line A X, are here of the ſame Value as at the Ingreſs V, the parallatic Time will be the ſame, *viz.* 7′ 46″ ½; therefore the obſerved Time of the Egreſs, or exterior Contact, will be 13′ *after Six* in the Morning.

4102. In order to find the Longitude of the Place Z from the Meridian of *London*, it muſt be remembered, that at the Beginning of the Tranſit in V, the Diſtance of the Meridian of *London* was 108° 4′ 45″ from that of Venus *Eaſtward*; and that while Venus moves from V to A, which is in Time 6ʰ 20′ 38″ (4068.) the Meridian of *London* will have paſſed through 95° 9′ 30″ of the celeſtial Equator, ſtill more eaſtward; the Diſtance thereof from that of Venus at the End of the Tranſit at A will be 203° 14′ 15″ *Eaſtward*, or 156° 45′ 45″ *Weſtward*. But the Longitude of Z is 88° 11′ 30″ Weſtward of that of

Venus alſo; therefore it is Eaſtward of *London* by 68° 34′ 15″.

4103. Again (by Fig. 4) it appears that both the Parallaxes conſpire to *accelerate the Egreſs* to an Obſerver on the *South Side* of Venus's Orbit, viewing it in the *Afternoon*. To determine the *Latitude* and *Longitude* of his Situation for making the Obſervation to the greateſt Advantage, we have *r* A Z = 52° 12′ 14″, and A Z = 83, as before; alſo *p* A M = 105° 46′ 45″, therefore *p* A *r* = 15° 46′ 45″; conſequently Z A *p* = 67° 58′ 59″. Alſo A *p* = 112° 32′ 58″. Therefore in the Triangle Z A *p*, we have known the two Sides A *p* and A Z, with the included Angle at A, by which is found the third Side Z *p* = 72° 43′ 27″, whoſe Complement 17° 16′ 33″ is the *South Latitude* ſought.

4104. By the ſame Triangle and Data, we find the Angle A *p* Z = 74° 30′, which is the Diſtance of the Meridian of the Obſerver from that of Venus, to the Eaſt. But the Meridian of Venus is then 156° 45′ 45″ eaſtward of *London* (4102) therefore the Meridian of the Obſerver at Z is 231° 15′ 45″ *eaſtward* from LONDON, or 128° 44′ 15″ to the *Weſt* of it.

4105. The ſame Angle A *p* Z turned into Time, gives 4ʰ 58′ from Noon, for the Time of the *central Egreſs* at that Place; and ſince all Things are the ſame here in the parallatic Triangle *v* A X, as in the foregoing Caſes, the Time of the Egreſs will be accelerated by the ſame Quantity, *viz.* 7′ 46″ ½, and therefore the Moment of the obſerved Contact at Y will be 4ʰ 50′ 14″ ½.

4106. As to a Calculation of the Latitudes and Longitudes of Places for the *interior Contacts* at G and B, it is no Ways neceſſary, for the Latitude will be nearly the ſame for both Contacts at the *Ingreſs* and *Egreſs*; and the Longitude for the interior Contact will be but 4° 30″ different from that of the external one, as it anſwers to the Time 18′ 36″ of paſſing through V G and B A; (Fig. 5. Plate II.) there-

therefore if 4° 30″ be added to the Longitudes already found for the exterior Contacts at the Ingreſs, and ſubducted from thoſe at the Egreſs, the Longitudes for the interior Contacts will be had.

## CHAP. VII.

### *A* Calculation *for thoſe* Places *where the apparent* Duration *of the* Transit *is a* Maximum, *or a* Minimum.

4107. THE Calculation of the *apparent Duration* of a Tranſit requires no Alteration of the Place of Obſervation; for in one and the ſame Place the *Ingreſs* may be view'd in one Hemiſphere, and the *Egreſs* in the other; and one ſhall be *accelerated* and the other *equally retarded*, by the whole Effect of Parallax. But this cannot be obtained in *one Hemiſphere only*, as may eaſily be obſerved in the Diagrams.

4108. But here we cannot avail ourſelves of both the *Maxima* at once, or greateſt Values of the Angle $v$ V X = 52° 12′ 14″, and the Side V X = 25″; but we can chuſe a Place of Obſervation with either of them ſingly, as will appear by the following Examples.

4109. If we make this Obſervation with the greateſt Zenith Diſtance of the Planet, *viz.* Z V = 83′ (ſo that V X = 25″) both at the *Ingreſs* in the *Evening*, and at the Egreſs the next Morning, 'tis plain the Meridian of the Obſerver muſt biſect the Arch of Duration; or the Complement of the Angle Z P V to a Semicircle muſt be equal to the Arch of half Duration, which is 47° 34′ 45″, conſequently the Meridian Z P of the Obſerver will be eaſtward of the Meridian of Venus P V by the Angle Z P V = 132°

$= 132^\circ\ 25'\ 15''$, and therefore it will be $24^\circ\ 20'\ 30''$, *eaſtward alſo of the Meridian of London.*

4110. The Time of Half-duration $3^h\ 10'\ 19''$ (4068) taken from XII, leaves $8^h\ 49'\ 41''$ for the Time of the Evening Obſervation of the Ingreſs; and of Courſe, the Time of Egreſs in the Morning following, will be $3^h\ 10'\ 19''$.

4111. The Latitude of the Place for this Obſervation is thus determined in the Triangle Z P V, there are given the Side V P $= 67^\circ\ 20'\ 4''$, the Side X V $= 83^\circ$ and the Angle at P, by which we find Z P $= 22^\circ\ 10'\ 37''$, whoſe Complement is $67^\circ\ 49'\ 23''$ = the Latitude ſought.

4112. In the ſame Triangle is found the Angle Z V P $= 16^\circ\ 18'\ 28''\ \frac{1}{2}$; to which add the Angle R V P $= 15^\circ\ 36'\ 26''$, the Sum is the Angle R V Z = X V $v = 31^\circ\ 54'\ 54''$. This Angle being known, and the Side V X $= 25''$ will give the Side X $v = 13'',2$; and the Side V $v = 21'',2$, from which we find $v$ Y $= 16''$. Then the Parallax in Longitude X $v = 13,''2$ gives $3'\ 15''$ of Time, and the Parallax in Latitude $v$Y $= 16''$ gives $3'\ 58''$, and the Sum of both is $7'\ 13''$, by which the *Ingreſs is accelerated.* And by the ſame Quantity the *Egreſs* will be *retarded* in the Morning, ſo that the *obſerved Duration* will be $14'\ 26''$ longer than the *central one*, and is a *Maximum.*

4113. The apparent Duration will be a *Minimum*, or the leaſt of all, in a Situation on the South Side of Venus's Orbit, the Longitude and Latitude of which are thus determined. It is evident (Fig. 3. Plate II.) that the Ingreſs being retarded in the Morning, and the Egreſs accelerated in the Afternoon (Fig. 4.) will make it neceſſary that the Meridian of the Place ſought ſhould here alſo *biſect the Arch of Duration*, or make the Angle in the Morning Z $p$ V = Z $p$ A $(= 47^\circ\ 34'\ 45'')$ in the Afternoon.

4114. From whence it will follow, that the Meridian Z $p$ of the Place will be $47^\circ\ 34'\ 45''$ Weſt of the Meridian

$p$ V paſſing through Venus at the Ingreſs, which added to 108° 4′ 45″ (the Diſtance of the Meridian of *London* eaſtward) will give 155° 32′ 30″, for the Situation of the Meridian of the Place, or its *Longitude*, *Weſt* from *London*.

4115. The Latitude of the Place is thus determined for the Maximum Angle X V $v$ = 52° 12′ 14″. In the Triangle Z $p$ V there are given the Angle Z $p$ V = 47° 34′ 45″, the Side $p$ V = 112° 39′ 56″, and the Angle $p$ V Z = 36° 35′ 48″, as we have ſhewn (4089.) by which we find the Side $p$ Z = 51° 31′ 26″, and therefore the Latitude of the Place is 38° 28′ 33″ ½ South.

4116. Alſo in the ſame Triangle we find the *Zenith-diſtance* of the Planet, or Side Z V = 75° 41′ 18″, which will give V X = 24″3 in the Triangle X V $v$; and ſince the Angle X V $v$ = 52° 12′ 14″, we ſhall find the Side X $v$ = 19″,2; and the Side V $v$ = 14″,89, and from thence we ſhall have $v$ Y = 11″,5.

4117. The Parallax of Longitude and Latitude together is X Y = 30″,7 which gives in Time of paſſing through it 7′ 36″, the double of which is 15′ 12″ is the Time by which the central Duration will be ſhortened at this Place; and is greater than that by which it will be lengthened in the North Obſervation (4112) by 46″; and is therefore ſo much the more advantageous.

4118. As the central Ingreſs will in this Place be at 8h 49′ 41″ in the *Morning*, the obſerved Ingreſs will be 7′ 36″ later, or at 8h 57′ 17″, and the obſerved Egreſs will be at 3h 2′ 43″ *Afternoon*, ſo that the obſerv'd Duration will be 6h 5′ 26″, a *Minimum*.

CHAP.

## CHAP. VIII.

*Of the* METHOD *of finding the* SUN'S PARALLAX *by obſerving the* leaſt Diſtance *of* VENUS *from the* SUN. *The* SUPERIORITY *of the* METHOD *of* PARALLAXES; *and of the* PRECISION *to which the* SUN'S PARALLAX *and* DISTANCE *of the* EARTH *and* PLANETS *from the* SUN, *may be thereby found.*

4119. ALL that has been hitherto ſaid relates to finding the Sun's Parallax by *parallatic Time*; but there is another Method which the Tranſit will furniſh for that Purpoſe, without any Regard to Time at all; this Method conſiſts in obſerving the *leaſt Diſtance* of Venus from the Sun's Center, when that leaſt Diſtance S *c* is a *Minimum* to an Obſerver on the *North Side*; and a *Maximum* when obſerved on the *South Side* of the Orbit of Venus.

4120. But as this is to be effected by Meaſurement of that Diſtance by Inſtruments properly conſtructed and ſkilfully applied; and then not by one ſingle and direct Meaſure of that leaſt Diſtance, but from a Reſult of many Meaſures taken from various Diſtances of the Centers of the Sun and Venus whilſt ſhe is traverſing his Diſk, or from many Obſervations made on the Differences of the *right Aſcenſion* of thoſe Centers during the Paſſage over the Diſk; I ſay, upon theſe Accounts, it will prove too critical and difficult an Affair for every common Obſerver to undertake.

4121. In a former Part of theſe Inſtitutes (3540, &c.) I have given the Conſtruction of an *Helioſtata* or *Celeſtial Clock*, by which the Image of the Sun is rendered *quieſcent*, or without Motion. By Means of a common reflecting Teleſcope this Image may be made 12, 15, or 18 Inches Diameter; ſo that every ſingle Second of a Degree becomes diſtinctly viſible to the Eye, and the Whole performed without

without any Difficulty or Trouble. Thoſe who are judicious in theſe Matters need not to be informed with how great Facility any Meaſures of the Diſtances of Venus from the Sun's Center may be taken during the Time of the Tranſit: This every one has acknowledged, to whom I have ſhewn the Effects of ſuch o'Clock.

4122. Indeed it would be more worth while to inſiſt upon this Method of meaſuring the leaſt Diſtance of the Centers, were it not in itſelf ſo deficient from anſwering the Purpoſe intended by it, *viz.* for eſtimating the Parallax of the Sun; for in the Inſtances we have given the Differences of the *central and obſerved leaſt Diſtances*, when reckoned in Defect and Exceſs, do in the Whole, make no more than 36″; and ſuppoſe they amounted to 40″, you could only find the Parallax within a *40th Part of the Whole*; which would give but little Satisfaction as it would leave ſo great Uncertainty in the Diſtance of the Sun. For Example, ſuppoſe the Sun's Parallax = 10″, and the Semidiameter of the Earth = 4000 Miles, then the Diſtance of the Sun would be 82500000 Miles; a 40th Part of this Number is 2062500; ſo that more than *two Million of Miles* would depend upon taking that leaſt Diſtance true *to one Second of a Degree.*

4123. The Method therefore, which principally, if not ſolely, deſerves Attention, is that by *parallatic Time*, both on Account of its Facility and the greateſt Preciſion we can ever hope to arrive at. As to the former, (to uſe Dr. HALLEY's Words) this great Problem is ſolved without much Labour,——and without any other Inſtruments than Teleſcopes, and good common Clocks; and without any other Qualifications in the Obſerver than Fidelity and Diligence, with a moderate Skill in Aſtronomy.

4124. Then as to the Preciſion, or Degree of Truth in the Determination, we have ſhewn there is 15′ 12″ in Exceſs (4117), and 14′ 26″ in Defect (4112), with regard to the Duration of the Tranſit from that obſerved at the Earth's

Earth's Center; ſo that by this Species of Obſervation, we have a Scope or Extent of 29′ 38″, or of 1778″ to eſtimate the Sun's Parallax by.

4125. But if we take the Differences of parallatic Time at the *Ingreſſes*, and *Egreſſes*, we ſhall have ſtill a larger Number of Seconds, *viz.* 31′ 6″ = 1866″. And the aforeſaid Dr. HALLEY has aſſured us, that he could very eaſily obſerve the internal Contacts of the Planet Mercury and the Sun, *without an Error of a ſingle Second of Time*; and from thence he concludes, that, in a Tranſit of Venus, theſe Contacts may be obſerved ſo accurately, that the *Sun's Parallax may thereby be determined* to a *ſmall Part of a Second*; and in another Place he affirms, the *ſaid Parallax may be known within a* 500*dth Part of the Whole.* And he was too great a Judge of ſuch Subjects to leave the leaſt Doubt of the Truth of what he has aſſerted.

4126. But after all, theſe Sums of Differences found at two different Places, depend upon precarious Principles, *viz.* the aſcertaining the *true Longitude of thoſe Places*; becauſe *every Minute's Difference in Longitude will make a Difference of four Seconds of Time.* Therefore I think the ſureſt Way will be to depend upon thoſe Obſervations of the *Duration* that are made in particular Places on the North, and on the South Side of the Orbit; where the Difference from the Central Duration will be found ſufficiently large, and very ſafe from Error; for 15′ 12″ give 912″, by which if you divide 82500000, the Quotient 90460 *will be the Number of Miles correſponding to each Second of Time.*

4127. Whence it appears, that if the Parallax of the Sun could be really determined to the 900*th Part of the Whole*, we ſhould ſtill be at the *Uncertainty of* 90000 *Miles in the Diſtance of the Sun from the Earth.* And yet this is perhaps a nearer Approach to it than we may ever be able to make; ſince in order to this, the Obſervation in all its Circumſtances, muſt *be juſt to a ſingle Second of Time.*

4128.

4128. Since all Effects are proportional to their Cauſes, it will follow, *that the calculated parallatic Difference of* 15′ 12″ *is to the aſſumed ſolar Parallax of* 10″ *as the obſerved parallatic Difference of Duration at the Time of the Tranſit is to the Parallax of the Sun*, as near the Truth as ſuch an Obſervation can give it. Thus for Example, ſuppoſe at the Place mentioned (4117) the Time of the Duration of the Tranſit to be found to be 6h 34′ 19″; this is greater than the central Duration 6h 20′ 38″, by 13′ 40″ = 821″; therefore as 15′ 12″ : 13′ 40″ : : 10″ : 9″ = *Sun's Parallax*, as truly as it can be obtained by the very beſt Obſervation.

4129. Again, ſuppoſe the Duration of the Paſſage be obſerved at the ſame Place to be 6h 36′ 36″ ½, this exceeds the central Duration by 15′ 57″ ½ = 957 ½, and therefore you ſay, as 912″ : 957 ½ : : 10″ : 10″ ½; ſo the Sun's Parallax will in ſuch a Caſe be 10 *Seconds and a Half*. But if the obſerved Duration there be juſt 6h 35′ 50″, which is 15′ 12″ longer than the central Duration, then the Parallax of the Sun will be known to be 10 *Seconds*.

4130. In the Triangle A B E (Fig. 4. Plate I.) ſince A E continues the ſame, it is plain the Side A B will be greater or leſſer in the *inverſe Ratio* of the Angle A B E, that is, the Diſtance of the Sun A B from the Earth E, will be *greater or leſs* than 82500000 Miles, in proportion as the ſolar Parallax A B E is found by Obſervation to be *leſs or greater than* 10 *Seconds*.

4131. Now the proportional Diſtances of the Planets from the Sun are known by the *periodical* Times reſulting from the Law of Gravity; and are thus ſtated, *viz.*

| For ☿ | ♀ | ☉ | ♂ | ♃ | ♄ |
|---|---|---|---|---|---|
| 38718 | 72333 | 100000 | 152369 | 520096 | 954006. |

4132. Therefore, if the Diſtance of one could be once known, thoſe of the other Planets will be known by the Rule of Proportion. Suppoſe the ſolar Parallax be found = 10″, the Diſtance of the Earth will 82500000 Miles; then ſay,

as 100000 : 825000000 : : 38710 : 32000000 = Diſtance of Mercury from the Sun in Miles; and ſo for the reſt.

4133. Not only the *Diſtances* of the Planets, but alſo their *Magnitudes* are known by the Parallax of the Sun. For ſince the *real Diameters* of Bodies are in the ſame Ratio of their *apparent Diameters*, or Semidiameters, and that of the Sun (at a Mean) being 15′ 30″ = 930″, and that of the Earth = (the *Sun's Parallax*) 10″; therefore the Sun's Diameter is to the Earth's Diameter as 930 to 10, or as as 93 to 1; and ſo the Sun's Magnitude is to that of the Earth, as 804357 to 1.

4134. By the laſt Tranſit 1761, the apparent Diameter of Venus was found to be 58″; therefore, if Venus were viewed at the Diſtance of the Sun, her apparent Diameter would be leſs in the Proportion of 100000 to 27667, that is, it would there be only 16″, but the Earth's Diameter at that Diſtance is 20″ (*viz.* double the Sun's Parallax) therefore, the true Diameter of the Earth is to that of Venus as 20 to 16, or as 5 to 4; and, of courſe, the Magnitude of the Earth is to that Venus, as 125 to 64, almoſt twice as large.

4135. Hence it follows, that the Magnitudes of the Sun and Planets will be greater or leſs in Compariſon of the Earth, as the Sun's Parallax at the next Tranſit ſhall be found leſs or greater than 10 Seconds. Therefore the final Determination of the *Dimenſions* of the PLANETARY SYSTEM depends on that Diſcovery.

CHAP.

## CHAP. IX.

*The* GEOGRAHY *of a* TRANSIT *illuſtrated by a* PRAXIS *on the* Terreſtrial GLOBE, *exhibiting all the* Phænomena *of the enſuing* TRANSIT *of* VENUS *to the various* INHABITANTS *of the* EARTH.

4136. THE GEOGRAPHY of a TRANSIT conſiſts in a general Repreſentation of all its Phaſes, in reſpect of the different Situation of the Inhabitants of the Earth, by the *terreſtrial Globe*; and to point out and diſtinguiſh all thoſe particular Places upon the Surface of the Earth, or Ocean, where the *Beginning, End, and Duration of the Tranſit*, may be obſerved to the greateſt Advantage poſſible, for the Diſcovery of the *ſolar Parallax*, and, in conſequence thereof, the *Dimenſions of the* SOLAR SYSTEM.

4137. In order to this curious and delightful Praxis, every Thing previouſly neceſſary has been already calculated; and to ſee the Reaſon thereof, one Thing only is to be conſidered, *viz.* that as the Sun enlightens one half of the Globe conſtantly, it is therefore always in the *Vertex or Zenith of that enlightened Hemiſphere*; and that the *Circle of Illumination* which terminates the light and dark Hemiſpheres, may be conſidered as a *ſolar Horizon*; and laſtly, that the Meridian in which the Sun is, is the *Eaſtern Horizon* in which he appears to be *riſing* to all the Inhabitants of the Earth under the *Weſtern Semicircle* of Illumination; and the ſame Meridian is the *Weſtern Horizon*, in which he appears to be *ſetting* to all the Parts of the Earth under the *Eaſtern Semicircle* of Illumination.

4138. Therefore, to rectify the Globe for the enſuing Tranſit of 1769, *June* 3d, the North Pole of the Globe muſt be elevated to the Sun's Declination, which is 22° 27′ 15″; (4062) then will this Point of the Sun's Declination on the South Part of the Braſs Meridian truly repreſent

the *Sun's Place* during the Time of the Tranſit, as being the *Zenith* of the illumined Hemiſphere.

4139. Then, ſince the Time of the *beginning* of the Tranſit is computed for LONDON, and is found to be at 7h 12′ 19″ in the Evening; (4069) therefore bring the Meridian of *London* to coincide with the general (or Braſs) Meridian, and holding the Globe faſt in that Poſition, ſet the *Hour-Index* to viih 12′ at Night; after this, turn the Globe Eaſtward till the ſaid Index points to the upper XII, and there let it reſt.

4140. Then it will be *Noon* to all thoſe Places which lie under the upper Semicircle of the general Meridian; and of Courſe they will ſee the beginning of the Tranſit, or firſt Contact of Venus, with the Limb of the Sun upon their Meridian, or at 12 at Noon. Theſe Countries you'll obſerve are only the *Weſtern Parts of North America*, the weſtern Parts of *Mexico*, by the Cape of *California*, and then over the vaſt *Weſtern Ocean* to 67° 33′ of South Latitude. The Sun will be vertical at that Time to a Place near Cape *St. Lucas*, the South Point of *California*.

4141. To all thoſe Places which lie under the Weſtern Semicircle of the Horizon of the Globe, the Sun will riſe with Venus in Contact on his Eaſtern Limb. Theſe are the *White Sea*, the moſt Northern Parts of *Siberia*, almoſt through the Middle of *Great Tartary*, the Northern Parts of *Japan*, and many of the Iſlands in the Great *South Sea*.

4142. Laſtly, all thoſe Places under the Eaſtern Edge of the Horizon will ſee the beginning of the Tranſit at *Sunſet*; viz. from the *White Sea*, the Northern Parts of *Ruſſia*, South Eaſt Parts of *Denmark*, the Southern Parts of the *Baltic Sea*, *Poland*, North Weſt Parts of *Germany*, South Eaſt Parts of *France*, the Eaſtern Coaſt of *Spain*, the middle Parts of *Morocco*, and all the Kingdoms upon the Weſtern Coaſts of AFRICA, and along the *Great Ocean* towards *South America*.

4143.

4143. From what has been ſaid, 'tis evident, that all Parts of the Earth *between the Weſtern Horizon* and the *Meridian*, ſee the beginning of the Tranſit in the *Morning* or *Forenoon*; and thoſe *between the Eaſtern Horizon and Meridian*, ſee it in the *Afternoon*; at different Hours according to their different Longitude or Diſtance from the Meridian.

4144. After the ſame Manner you proceed for the *middle of the Tranſit*, which will be at 10h 22′ ½ at Night; therefore bring LONDON to the Meridian, where hold the Globe faſt, and ſet the Hour Hand to 10h 22′ ½, and then turn it till the Index comes to the upper XII. Then, in the Weſtern Horizon, you ſee all the Countries who obſerve the Planet in the middle of her Path upon the Sun's Diſk at *his riſing*; under the Meridian are all thoſe Places who have that Phaſe at Noon Day; and in the *Eaſtern Horizon* are all thoſe which ſee the ſame at *Sun-ſet*. Alſo in this Poſition of the Globe you have in one View all the Places who view the middle of the Tranſit in the *Forenoon*, and in the *Afternoon*, at every Hour.

4145. The *End* of the TRANSIT will be at 1h 33′ in the Morning of *June* 4th; therefore placing LONDON under the Meridian, and the Hour Hand at the ſame Time to 1h 33′, turn the Globe eaſtwards till the Index comes to 12 at Noon, and there let it reſt. Then in the Weſtern Horizon are all the Places who ſee the *Sun-riſing* at the End of the Tranſit; thoſe under the Meridian ſee the laſt Contact at *Noon*; and the ſame Phaſe appears at *Sun-ſet* to all who are in the Eaſtern Semicircle of the Horizon.

4146. From what has been ſaid it will follow, that if when the Globe is rectified for the beginning of the Tranſit as directed (4139) you draw a Circle quite round the Globe by the interior Edge of the Horizon with a piece of Chalk or Crayon, and afterwards ſuch another Circle, when the Globe is rectified for the End of the Tranſit (as in the laſt) then will thoſe two great Circles interſect each

other in Points 31° 29′ distant from the Poles of the World, and divide the Surface of the Globe into four unequal Parts, or Gores.

4147. Then in that Gore, which is contained between the two eastern Semicircles, which shew the Sun *setting* at the Beginning and End of the Transit, you will see those Places *where the beginning of the Transit will be seen, but not the End*; viz. the Eastern Parts of *North America*, all the *West Indies*, and *South America*; the Western Parts of *Europe* to the Latitude of 65 Degrees; the Western Parts of *Africa*, and all the *Atlantic Ocean*.

4148. In the Quarter or Gore opposite to this, you observe those Places only *who see the End of the Transit but not the Beginning of it*; because at the Beginning, this Gore was wholly in the nether or dark Hemisphere. The Countries herein comprehended are *Russia*, and the greatest Part of the Continent and Islands of *Asia*, *New Holland*, *Zealand*, &c.

4149. In that Quarter bounded by the two Semicircles, of which one shews the Sun *rising* at the beginning of the Transit, and the other shews him *setting* at the End of it, are contained all those Parts of the World who see the *Whole of the Transit*; because they are in the illumined Hemisphere, or above the Horizon, the whole Duration of the Transit. This Quarter of the Globe is almost wholly possessed by the *Pacific Ocean*, or Great *South Sea*, and its numerous *Islands*. In the Northern Parts of it are contained the most Part of *Great Tartary*, *Nova Zembla*, all *Lapland*, *Baffin's Bay*, *Hudson's Bay*, and all the unknown Part of *North-West America*.

4150. In the Quarter opposite to this, you find the Situation of all those Countries whose Inhabitants see *nothing at all of the Transit*. Such are the *midland Parts of Europe*, all the *Mediterranean*, all the *Arabia's*, and part of *Persia*, and nearly the whole *Continent of Africa*, with the *vast Ocean* to the South thereof; for these Parts will be under the Horizon

Horizon during the whole Paſſage of Venus over the Sun's Disk.

4151. Since the Declination of the Sun 22° 27′ is ſo near the greateſt Declination 23° 30′, it will follow, that a Tract about the North Pole, nearly equal in Extent to the whole *Frigid Zone*, will at that Time of the Year have the Sun conſtantly above the Horizon; the Inhabitants of one Part of this Zone will have a Motion in the ſame Direction of the Planet Venus, *viz.* from Eaſt to Weſt; and this will cauſe the Duration of the Tranſit to be longer to them than the central one; and the Inhabitants of the oppoſite Parts of the ſame Parallel, as they move in a contrary Direction, will obſerve the Duration to be ſhorter than that ſeen from the Earth's Center.

4152. But as we have already determined *ſix different Places* where the Differences by parallatic Time are the *greateſt and leaſt poſſible*, with regard to the *Ingreſs*, *Egreſs*, or *whole Duration*, we ſhall next point out the Situation of thoſe Places upon the Surface of the Globe by means of their given Longitudes and Latitudes.

4153. We have ſhewn that the Place where the Ingreſs is moſt of all accelerated, is in North Latitude 51° 28′, and and Longitude 6′ Eaſt from *London*; and that points out the City of LONDON itſelf for the beſt Situation to obſerve that Phaſe (4090).

4154. The retarded Ingreſs is a *Maximum* in South Latitude 43° 30′, and Longitude Weſt from *London* 162° 49′; this Longitude brought to the Braſs Meridian will give the Point under the 43° 30′ for the Scite of the Place required, which is in the ſouthern Part of the *Pacific Ocean.* (4091).

4155. The Place where the retarded Egreſs is ſeen to the beſt Advantage has Latitude 23° North, and Longitude Eaſt of *London* 68° 34′ (4097), therefore bring this Longitude to the general Meridian, and under the 23° of Latitude you will find the Place to be a little above *Surat* on the Gulph of *Cambay*, near the *Tropic of Cancer*.

4156.

4156. And the Place where this Phaſe is moſt of all accelerated is in *South Latitude* 17° 16′, and 128° 44′ Weſt of *London*. (4103). This will give the Situation of the Obſerver in the *Pacific Ocean*.

4157. Then again, we have ſhewn (4112) that the Place where the Duration will be obſerved a Maximum is in Latitude 67° 49′ North, and Longitude 24° 20′ Eaſt of *London*; this therefore will be found a little above *Tornea*, in *Swediſh Lapland*.

4158. And laſtly, the Place where the Time of the Tranſit will be obſerved a *Minimum*, is in South Latitude 38° 29′, and Weſt Longitude 155° 32′; which determines the Place in the *Pacific Ocean*, or Great *South Sea*. (4118).

4159. It is further to be remarked, that tho' the ſix Places above determined are indeed the beſt that can be, yet a Diſtance of many Leagues round either, will make but a trifling Difference in parallatic Time; and therefore the Obſerver may look upon theſe Places only as *Indices* of what Parts of the World will be moſt proper for his Station and Obſervations. Thus 50, 80, or 100 Miles round LONDON, will be nearly of the ſame Advantage as the City itſelf. All the *Mogul's* Empire about *Surat* and *Cambay* will do for Obſervations there; and over all *Norway* and *Lapland*, the *Northern Obſervations* will be equally good.

4160. On the other Side the Line we have the three Stations near the ſame Part of the *Great Ocean*, and all beyond the Iſlands ſo numerous in leſſer Latitudes, and other Parts of the ſame Ocean, which nevertheleſs will be found good Situations for thoſe *Southern Obſervations*.

4161. As we have been ſo large and particular in the *geographical Praxis* by the Globe, it will be quite unneceſſary to give a *geographical Map of the Tranſit*, as it is much more difficult and leſs perſpicuous and pleaſant to ſtudy the Geography of the Tranſit that Way, than by a *Tranſit Globe*, with proper Circles drawn upon it, and all the

the Diviſions or Gores properly diſtinguiſhed by different Colours where the various and different Phaſes and Quantities of the Tranſit will be obſerved over the Face of the whole Earth.

4162. I cannot leave this Subject without remarking an Error in all the Maps of the Tranſit I have ſeen, *viz.* that it is aſſerted by their Authors, *that to all the Space contained between the two Semicircles mentioned in* (4149) *the Whole of the* Tranſit will be viſible; *and to all the Space of the oppoſite Gore (in* 4150), the Whole of the Tranſit will be inviſible. Now, that this is not true will appear by rectifying the Globe for the *middle of the Tranſit* as directed in (4144), and drawing a Circle upon the Globe quite round the Horizon, for then it will be ſeen that the Gore of perpetual Apparition (4149), is not wholly in the upper and illumined Hemiſphere, but a Part at the *North End* will be cut off by this Circle, and conſequently muſt be below the Horizon at the middle of the Tranſit, and to them therefore that Phaſe will be inviſible.

4163. On the other Hand, a Part of the Gore mentioned in (4150) will be in the enlightened Hemiſphere at the middle of the Tranſit, and therefore to ſome Part of this Space, ſome Part of the Tranſit will be viſible. This reſcinded Part of either Gore is in Form of an *Iſoceles Triangle* whoſe Altitude is 9 Degrees, and its Baſe about 19°; and of courſe not ſo very inconſiderable as to be wholly neglected in an accurate Explication of the GEOGRAPHY of a TRANSIT. But we ſhall further conſider this Subject in a Chapter on the *Tranſit Globe*.

## CHAP. X.

*The* NATURE *of a* TRANSIT *and its various* Phænomena *explained, by an* Orthographical PROJECTION *of the* EARTH, SUN, *and Planet, upon a* Perſpective PLANE, *and a general* COMPUTATION *of* parallatic TIME *from thence.*

4164. THERE is another Method of treating a Tranſit, which can by no Means be omitted in a Syſtem of Inſtitutions of this Kind; the Knowledge of which no ingenious Student in Aſtronomy can be ſuppoſed to diſpenſe with; which is in itſelf moſt elegant and geometrical; and laſtly, a Method which gives the general Rationale of all others; it conſiſts in an *Orthographical Projection* of the Earth, the Sun, and Planet, upon a perſpective Plane paſſing through the Center of the Planet, and perpendicular to the Axis of a Cone of Rays, from a Point of Sight in the Center of the Sun.

4165. The Principles of this Projection, applied to *Solar and Lunar Eclipſes*, have been already largely explained in theſe Inſtitutes (1721, *&c.*); they are equally neceſſary for a true Repreſentation of a Tranſit *in Plano*, as will appear from a due Conſideration of Fig. 4. Plate I. wherein the Diameters, or rather Semidiameters, of the Sun, the Earth, and the Planet, are evidently as the Angles under which they appear as viewed from the Earth, Sun, or Planet. It has alſo been ſhewn, that the Parallax B C of the Planet V by two Obſervers, one at the Earth's Center E, and the other upon its Surface at A, will be the Angle B V C, or A E; but the ſame Parallax B C will be ſeen by the Obſerver at A only, under the Angle B A C.

4166. But this Angle B A C depends upon the Planet's Diſtance from the Sun, as it increaſeth and decreaſeth therewith; alſo it is well known, that the Perſpective of any

Object

Object upon a Plane, will increaſe and decreaſe with the Diſtance of that Plane from the Eye. From all which it follows, that if the Eye be ſuppoſed to be placed in the Center of the Sun at S, and a perſpective Plane at the Orbit of the Planet P V, perpendicular to the viſual Line S P E, then a Circle deſcribed upon the Center P of ſuch a Plane, with the Radius of 945 equal Parts or Seconds, will repreſent the Disk of the Sun, for ſo many are contained in his Semidiameter S D = 15′ 45″, as viewed from the Earth at E. (Fig. 4. Plate I.)

4167. Again, to an Eye at the Sun in B, the perſpective of the Earth's Semidiameter A E will be equal to Q V upon the Plane. But A E is to Q V, as the Parallax of the Planet A V E to the Parallax of the Planet from the Sun Q A V. That is, in our preſent Caſe of the Planet *Venus*, we have A E : Q V : : 35″ : 25″ (4078), conſequently the Semidiameter of the Sun and Earth upon that perſpective Plane, will be as 945 to 25.

4168. Therefore, with a Radius S E = 945″ (taken from a Scale of equal Parts) deſcribe on the Center S the Arch E F, to repreſent a Part of the Limb of the ſolar Disk; and upon the ſame Center S with a Radius S *a* = 25″ (from the ſame Scale) deſcribe the ſmall Circle *a f b g*, to repreſent the Perſpective of the Earth's illumined Hemiſphere; then will *a b* repreſent that Meridian in which the Sun is at S, *a g b* is the eaſtern Semicircle of Illumination, and *a f b* is the weſtern Half. (See Fig. 1. Plate III.)

4169. Let the Line C V be half the Tranſit Line A V in the Diagrams of Plate II; and becauſe the Conjunction at D happens at 10 o'Clock (4057), therefore if you take 242″ from the Scale, and placing one Foot of the Compaſſes in D, croſs the ſaid Line with the other Point towards the right Hand, it will give the Hour of IX; from thence you ſet the ſame Diſtance to VIII. and then to VII. and thus

the Path of Venus is divided into Hours, each of which is ſubdivided into Minutes ſufficiently diſtinct to the Eye; ſo that the Place of Venus is ſeen for *every Minute of her Paſſage.*

4170. Let S C be perpendicular to the Path, as in the other Figures, and let V, E, G, be the Points in which the Planet is at the external Contact, central Ingreſs, and interior Contact; the Time of the former is $7^h\ 12'\ 19''$ (4069), and of the latter, $7^h\ 30'\ 55''$ (4070), as ſeen from the Earth's Center S. And taking the Extent from S to V in the Compaſſes, place one Point in V, and with the other deſcribe a ſmall Arch *i h*; then will all the Inhabitants of the illumined Diſk over which the Arch *i h* paſſes, ſee the beginning of the Ingreſs at the ſame time it would be viewed from the Earth's Center. In the ſame Manner you proceed for the central Ingreſs, and internal Contact at E and G.

4171. The ſame Extent or Line S V being taken in the Compaſſes, one Foot of which being placed on any particular Point of the Earth's enlightened Surface, the other will fall upon the Minute of Time in the Path V, at which the external Contact will happen to the Inhabitants of that Place.

4172. It is evident from this Operation, that to all Parts of the Earth's Diſk which lie beyond the Arch *c h*, the Time of the beginning of the Tranſit will be *later* than the central Time; and lateſt of all at the remoteſt Point *r* of that farther Hemiſphere.

4173. On the other Hand, one Foot of the Compaſſes being placed upon any Part of the near Hemiſphere, the other will fall upon that Minute in the Tranſit Line at which the beginning of the Tranſit happens in that Place, which of courſe will be *before the Time at the Center*, or in the Arch *i h*; and the moſt early beginning will be to thoſe who live at *o*, in the Part nearly oppoſite to *r*.

4174. If the ſame Conſtruction be made for the Planet's Egreſs from the ſolar Diſk, then the Moments of the Phaſes of the beginning, central, and total Egreſs, will, in like Manner, be ſeen or found for all Places upon the Earth's Diſk (*a f b g*).

4175. In all that has been hitherto ſaid, the Reader will obſerve, that the Triangles S C V, *&c.* and all the Sides and Angles thereof, are of the ſame Quantity as thoſe leſſer ones mentioned (4065), but here the Projection is large enough to give a diſtinct Idea of the Manner in which the Moments of Time for the various Phaſes of the Tranſit are to be aſcertained for the ſeveral Parts of the Earth's Surface, where it can in any Part be ſeen.

4176. If the projected Diſk of the Earth (*a f b g*) be three Inches in Diameter, (as in Fig. 4.) the Line S V will be 4 Feet 10 Inches; then that Diſtance taken in the *Beam Compaſs*, and the Point placed in *v* and *r*, the Pencil will mark the Minute on each Side the Point V in the Path, which denote the earlieſt and lateſt Moments of the beginning of this Sort of Eclipſe; then afterwards place the Point of the Compaſs upon each Minute between theſe two ſucceſſively, and upon them, as Centers, ſtrike circular Arches upon the Earth's Surface with the Pencil; theſe Arches will divide the then enlightened Surface in ſuch Manner as to ſhew all the Places where the beginning of the Tranſit differs by one Minute of Time. And thus you divide the Earth's Surface for the Minutes of the Difference of Time at the end of the Tranſit.

4017. From what has been ſaid, it will be eaſy for the young Student in Aſtronomy to underſtand how the ſame Minute Arches may be drawn upon the Surface of an artificial Globe itſelf, of 3, 6, or 9 Inches Diameter, by placing it in a circular Hole of a board, or Plane, with half a Meridian to move upon the end of the deſcribent Line, and to be ſet at right Angles to it.

4178. It now remains to assign the Seat or Perspective of any proposed Place upon the Earth's projected Disk; for this Purpose the Latitude and Longitude from the Meridian *a b* must be known; as also the Position of the Plane of the Meridian passing through the Place, and the Elevation thereof above the primitive Circle, or Plane of Projection. To find the two last Requisites proceed thus; let A B be the Meridian of the enlightened Hemisphere passing through the Sun S in the Center; B P the Elevation of the North Pole P, stereographically projected; and suppose C P E D the Meridian of the Place making the Angle E P S or C P B = 35°, or given Longitude, from the first Meridian A B. Then in the right-angled spherical Triangle B P C, there are known the Side B P = 22° 27′ the Sun's Declination, and the Angle C P B = 35°; by which you find the Side B C = A D = 15°, and the Angle B C P = 58°. (See Fig. 2. Plate III.)

4179. Then take S B in the Compasses and make it a Radius on the Line of Sines upon the Sector; and then take the parallel Distance from 32 to 32, (the Compleplement of 58°) and set it from S to L; so will S L be the semi-conjugate Axis, and S C the semi-transverse of the elliptic Meridian passing thro' all Places of 35° Longitude from the Meridian A B; and may be drawn as directed (785.)

4180. Let the Latitude of the Place be 40°; the elliptic Parallel of that Latitude is therefore next to be drawn. In order to do that, you proceed thus, by the orthographical Projection; let E P Q *p* be the solar Meridian of the illumined Hemisphere placed directly before the Eye (Fig. 3. Plate III.) its Diameter A B; Center S; E Q the Equator; P *p* the Axis; C D the Diameter of the Parallel of 40° = E C, and its Center, N; also *c d* is the same Parallel of South Latitude, and its Center *n*; and ⊙ the Sun in the Zenith.

4181.

4181. Then right Lines perpendicular to A B, paſſing thro' the Points P, N, C, ✳, *n c* will project thoſe Points into the Points O, R, ♈, S, *r*, *t*, in the Axis A B; and the Radius S B ſet from 90 to 90 in the Sines upon the Sector, you will have the parallel Diſtance from 67° 33′ upon the ſame Sines equal to S O, for the projected Pole O.

4182. The projected Center of the parallel is R, which is therefore the Center of the Ellipſis to be drawn; the longeſt Diameter of the Ellipſis will be equal to the Diameter C D of the Parallel, or the Semi-tranſverſe is equal to C N.

4183. The Projection of the Point C of the Parallel in the Meridian is T, and its Diſtance from the Center S is S T, equal to the parallel Diſtance of the Sines 17° 33′ on the ſectoral Lines; that is, S T is the Sine of the Arch ✳ C = E C − E ✳, or Difference between the Latitude of the Parallel and the Sun's Declination. But S *t* is the Sine of their Sum, *viz.* *c* E + E ✳, for the ſame Parallel in South Latitude. From hence it is evident, that T R (= *t r*) is the ſemi-conjugate Axis of the required Ellipſis, and therefore having both the Axes, the Ellipſis is drawn as directed in (785)

4184. Therefore (in Fig. 4.) let A B be the Meridian of XII. on the illumined Disk of the Earth, and let K M L be the Meridian of the given Place in 35° Weſt Longitude of A B; (4179.) alſo let C T D be the Parallel of 40° the Latitude of the Place propoſed, which is now determined upon the Disk in the Point of Interſection M of the ſaid Meridian and Parallel.

4185. Then taking S V in the Compaſſes (Fig. 1.) and placing one Foot in the Point *m* upon the Earth's Disk, extend the other to the beginning of the Path towards V, and it will fall upon the Minute of Time at which the Contact at V will happen to a Spectator on the Earth in Latitude 40° and Weſt Longitude 35°. If you take the

Line

Line S G in the Compaſſes and apply it as before, it will ſhew the Minute when the internal Contact will happen at the ſame Place.

4186. If a Calculation to determine the Point M be more eligible, you proceed thus: As Radius is to the Sine of the Co-Latitude C O, ſo is F S to C R. Alſo C R is to M V as Radius to the Sine of the Longitude 40° = Arch M T; therefore by Compoſition of Ratios, we have $\frac{s\,C\,O \times s\,M\,T \times F\,S}{R^2} = M\,V$.

4187. Again, Radius is to the Sine of C M = 55°, as R T is to W M = R V; and S R − R V = S V; then having the Length of S V and V M, the Point M is thereby aſcertained upon the Disk.

For Example, the Sine of C O = 50° − 9.884254
Sine of M T = 35° − 9.758591
F S = 25 − 1.397940

From the Product 21.040785
Subtract R² — 20.

There remains the Value of M V = 10″,98 − 1.040785

4188. Then by the Analogies above (wrought upon the Sliding Rule) you will find S R = 16″; and Radius is to the Sine of the Arch ⊙ C (Fig. 3.) as F S is to S T = 7″,6. Then S R − S T = 8,″4 = T R; and laſtly R : *s* C M = 55° : : T R : M W = R V = 6″,88. Then S R − R V = S V = 9″,12. Having thus found V M and V S in Numbers, the Point M is given upon the Disk by the plain Scale and Compaſſes.

CHAP.

## CHAP. XI.

*Of the* ANALOGY *and* CORRESPONDENCE *between the* ORTHOGRAPHICAL PROJECTION, *the* METHOD *by* DIAGRAMS, *and the* Praxis *by the* GLOBE, *in regard to the various* PHASES *of a* TRANSIT.

4189. I Shall now point out the Analogy and Correſpondence there is between this orthographical PROJECTION and the Method by DIAGRAMS in Plate II. and the Praxis by the *Terreſtrial* GLOBE in Chap. IX. In order that this may be more evidently made out, it muſt be firſt obſerved, that in ſuch a Projection as the preſent one, the Vertex or Zenith of the Place under the Sun is the Center thereof, or the Point S, and that all the vertical Circles S Z being at Right Angles to the Plane of the Primitive, are projected into right Lines S Z, and that the Zenith Diſtance of any Place M is S M, and whoſe Meaſure is on the Line of Sines, S Z being Radius.

4190. Therefore ſince Radius is to the horizontal Parallax S Z, as the Sine of the Zenith Diſtance is to the Parallax S M pertaining to the Place M, (4021). It is plain that the Line S M in this Projection correſponds to the Line V X in the Diagrams; and again, let the Diameter P Q be the Axis of Venus's Orbit upon the Disk, and from the Point M draw M *v* perpendicular to it, then will the Parallax of Altitude M S be thereby reſolved into two others, *viz.* S *v*, which is the Parallax of Latitude from the Orbit, and the other is M *v* parallel to the Orbit, and therefore is the Parallax of Longitude in the apparent Path. Conſequently S *v* and M *v* do here anſwer to V *v* and X *v* in thoſe Diagrams, therefore the Triangle S *v* M and V *v* X are analogous, or the ſame Thing in different Views.

4191. If N repreſent a Place in the ſame Parallel of Latitude, and in the ſame Longitude, but on the other Side or Eaſtward of the Meridian A B, then drawing the Perpendicular N *s*, the Line N *s* will be the Parallax in the Orbit; and the Line S *s* is the Parallax of Latitude; if N *s* be continued it will cut S *y* in *w*; and the Triangle S *w* N will anſwer to the parallatic Triangle V Y X in the Diagrams, and *s w* is the efficacious Part of the Parallax of the Latitude S *s*, becauſe the Angle N S *s* = X V *v*; and S *s w*, or P S *y* = V *v* Y.

4192. If M be the Site of a Place in the Weſtern Hemiſphere where the Tranſit begins, and N that in the Eaſtern Hemiſphere where it ends, and T M = T N; then S *v* will be the Parallax of Latitude at the beginning, and S *s* that at the end; and ſo the apparent Path *m n* of Venus to that Place will be inclined to the true or central Path M N in the Diagrams, and not parallel, as there repreſented.

4193. Draw *a s e* at right Angles to S P, and let E be a Place near the Weſtern Horizon in the Parallel of the Sun's Declination where the Tranſit appears to begin, and S its Place when it ends; then draw E *b* perpendicular to S *a*, and it will be the Latitude or Diſtance of the apparent Path, and the true one at the beginning of the Tranſit; but at the End, they both coincide. And what is more, all Places (circumſtanced as above) between North Latitude E and South Latitude *a*, will be on the *North Side* of the Orbit at the beginning, and on the *South Side* at the end of the Tranſit; and ſo the apparent Path *m n*, (in the Diagrams) will interſect the central one M N, or lie acroſs it.

4194. It is further evident, that to every Place N in the eaſtern Hemiſphere P *e* Q, the Parallax of Longitude N *s* conſpiring with the Motion of Venus, will accelerate every Phaſe of the Tranſit; and if the Place be on the North Side or Hemiſphere *a* P *e*, then alſo the Parallel of Latitude

*s w* will be directed the ſame Way, ſo the Phaſe will be accelerated by the Sum N *w* of both Parallaxes, conſentaneous to (4051.)

4195. But if the Place be on the ſouthern Side of the Orbit, or Hemiſphere, *a* Q *e*, then the Parallax of Latitude has a contrary Tendency to that of Longitude, and the Phaſe of the Tranſit is accelerated by their Difference ſo long as that is poſitive, that is, ſo long as the Parallax of Longitude is greater than that of Latitude.

4126. For ſince after you paſs the Orbit of Venus *a* S *e*, the Parallax of Latitude becomes negative and increaſing, while that of Longitude conſtantly decreaſes, there muſt be ſome Point *N* in every Secondary P N Q, where the negative Parallax of Latitude will be equal to the Parallel of Longitude *N g*. And any Place to the Southward of that, or towards Q, will have the Phaſe retarded by the ſaid Differences of Parallaxes.

4197. In the Points P and Q, where the Longitude vaniſhes, and where the Effect of the Parallax of Latitude is greateſt of all, the Value of this Parallax will be expreſſed by the Line Q Y, or Tangent of the Angle Q S Y compared with Radius Q S. But the greateſt Effect of the Parallax of Longitude is at *a*, or *e*, and is the horizontal Parallax itſelf, *viz.* S *a* or S *e* = 25″,14, (4077) therefore the greateſt Effect of the Parallax of Longitude is to that of Latitude, as S *e* or Q S to Q Y, or as Radius to the Tangent of the Angle Q S Y = 37° 48′, in the preſent Caſe.

4198. An Obſerver at any Place M in the Weſtern Hemiſphere P *a* Q, and on the North Side of the Orbit *a s e*, will obſerve the Phaſes of the Tranſit accelerated, but it will be only by the Difference of Parallaxes; for the Parallax of Longitude M *v* retards, and that of Latitude *v u* accelerates them, and the Difference M *u* is poſitive; but that vaniſhes when the Place M is in

the Line S Y; and afterwards, in all the Hemisphere *b r i*, both the Parallaxes conspire to retard the Phænomena of the Transit. But in the Parts of the Disk between *y* S and *a* S, the Retardation is effected by the Differences of the two Parallaxes, that of Longitude being greatest and negative. In all Parts between *a* S and S Q the Sum of the Parallaxes retard the Times. Lastly, from S Q to S Y, they are again retarded by the Difference; since there the Parallax of Latitude prevails over that of Longitude, and is contrary to the Motion of Venus.

4199. The Line *i h* (in Fig. 1.) which divides the Surface of the Earth's Disk into the two Parts *i o h* and *i r h*, in the present Construction of Fig. 4. coincides with the Line *y* Y; and the Angle *o* S P is the Maximum Angle, and equal to the Angle Z V R, or X V *v*, in the Diagrams of Plate II. Also the Angle *i* S P is equal to the Angle *v* V Y there. If therefore the Line *o k* be drawn parallel to *a e*, it will cut P S in *l*, and the Part *o l* will be the horizontal Parallax of Longitude, and *l k* that resulting from the Parallax of Latitude S *l*; both accelerating the beginning of the Transit in the greatest Degree possible.

4200. In order to find the Quantity of this greatest Acceleration (and that of any other) the two Scales A B and C D may be made, as in Fig. 5. The first A B is equal to the Radius of the Disk, and is therefore divided into 25″,14 being equal to the horizontal Parallax of Venus from the Sun (4077); upon this the *Number of Seconds of a Degree* in any Parallax is measured. And the Scale C D is divided into *six Minutes of Time*, and is equal to 24,2 Seconds of Motion in the Scale A B, as is evident from (4067).

4201. Thus for Example, take *o l* in the Compasses, and measure it upon the Scale A B, and it will be equal to 20″ of Motion; upon the Scale C D it will be equal to 5′ of Time in which that Motion is performed. Then *l k*

(the

(the Effect of Latitude) measured upon A B is near 12″ of Motion; upon C D it is near 3′ of Time; so that both these *Maxima* together make almost 32″ of Motion, or nearly 8′ of Time; which is the greatest Quantity by which the beginning of the Transit can be accelerated. By considering the Position of the Meridian O *o*, and that of the Point *o*, they will be found near to those of *London*, as will appear by rectifying the Globe as taught in (1453).

4202. Hence also it appears that the greatest Effect of Latitude by which the Phase can be accelerated is P *y*, and equal to Q Y by which it will be retarded; and if measured upon A B will be found 19″ ½; and upon C D it will give about 4′ 50″ of Time.

4203. I shall only remark further, that in the Semi-Disk of Acceleration *i o h*, there is an entire Quadrant between *e* S and P S, where the Acceleration is caused by the *Sum* of the Parallaxes. But from *h* to *e*, and from P to *i*, it is the Effect of their *Difference*; between *h* S and S *e* the Parallax of Longitude is positive, and prevails over the negative Effect of Latitude; but between S P and S *i*, the Parallax of Longitude is negative, and less than that of Latitude which is affirmative. And the same holds good with regard to the Retardation in the Semi-disk *i r h*, as we have in Part already observed.

4204. From the Whole of this Process it appears, that this orthographic Projection is nothing but the Praxis on the Globe, as directed for exhibiting the various Phænomena of the beginning of the Transit (4139), here projected upon a Plane; and by adapting this Projection to the *Middle* and *End* of the Transit, it will equally express, or rather give the Rationale of all the Affections of those Phases which were deduced from the Globe itself; and this I shall recommend to the Reader as a Practice that will much facilitate his Conception of these intricate Matters, and give them a lasting Impression on his Mind.

CHAP.

## CHAP. XII.

*Of the* TABLES *neceſſary for computing the* TIME *in which any given* QUANTITY *of* MOTION *is performed; and the* MOTION *produced in any given* TIME; *illuſtrated by* EXAMPLES *of their* USE *in the Doctrine of* TRANSITS, *and other* PARTS *of* ASTRONOMY.

4205. THERE is no Part of Aſtronomy which requires the Uſe of ſuch Tables as ſerve to convert *Motion into Time*, and *Time into Motion*, more than that which is converſant about Tranſits, as is evident from all the preceding Calculations on this Subject. We have already explained the aſtronomical Principles of celeſtial Motions, and the chronological Doctrine of Time in the Body of theſe Inſtitutes. It remains now to ſupply thoſe Tables by which Motion and Time are commutable into each other, and then to explain their Uſes by an Example or two.

4206. The firſt Table is that by which the Motion of the Earth's Equator is expreſſed in Time; or rather, it ſhews the Time in which any Number of *Degrees*, *Minutes*, *Seconds*, &c. of the Equator paſs under the Meridian of a Place. This Motion is at the Rate of 15° per Hour, or one Degree in 4′, becauſe the whole 360 revolve in 24 Hours, and $24 \times 15 = 360$.

TABLE

## TABLE I.

*In which the Degrees, Minutes, and Seconds of* LONGITUDE, *or* right ASCENSION, *are expreſſed in Hours, Minutes, and Seconds of* TIME.

| Deg. / Min. / Sec. | H. M. / M. S. / S. T. | Deg. / Min. / Sec. | H. M. / M. S. / S. T. | Degrees. | Hours. |
|---|---|---|---|---|---|
| 1 | 0 4 | 31 | 2 4 | 70 | 4 40 |
| 2 | 0 8 | 32 | 2 8 | 80 | 5 20 |
| 3 | 0 12 | 33 | 2 12 | 90 | 6 0 |
| 4 | 0 16 | 34 | 2 16 | 100 | 6 40 |
| 5 | 0 20 | 35 | 2 20 | 110 | 7 20 |
| 6 | 0 24 | 36 | 2 24 | 120 | 8 0 |
| 7 | 0 28 | 37 | 2 28 | 130 | 8 40 |
| 8 | 0 32 | 38 | 2 32 | 140 | 9 20 |
| 9 | 0 36 | 39 | 2 36 | 150 | 10 0 |
| 10 | 0 40 | 40 | 2 40 | 160 | 10 40 |
| 11 | 0 44 | 41 | 2 44 | 170 | 11 20 |
| 12 | 0 48 | 42 | 2 48 | 180 | 12 0 |
| 13 | 0 52 | 43 | 2 52 | 190 | 12 40 |
| 14 | 0 56 | 44 | 2 56 | 200 | 13 20 |
| 15 | 1 0 | 45 | 3 0 | 210 | 14 0 |
| 16 | 1 4 | 46 | 3 4 | 220 | 14 40 |
| 17 | 1 8 | 47 | 3 8 | 230 | 15 20 |
| 18 | 1 12 | 48 | 3 12 | 240 | 16 0 |
| 19 | 1 16 | 49 | 3 16 | 250 | 16 40 |
| 20 | 1 20 | 50 | 3 20 | 260 | 17 20 |
| 21 | 1 24 | 51 | 3 24 | 270 | 18 0 |
| 22 | 1 28 | 52 | 3 28 | 280 | 18 40 |
| 23 | 1 32 | 53 | 3 32 | 290 | 19 20 |
| 24 | 1 36 | 54 | 3 36 | 300 | 20 0 |
| 25 | 1 40 | 55 | 3 40 | 310 | 20 40 |
| 26 | 1 44 | 56 | 3 44 | 320 | 21 20 |
| 27 | 1 48 | 57 | 3 48 | 330 | 22 0 |
| 28 | 1 52 | 58 | 3 52 | 340 | 22 40 |
| 29 | 1 56 | 59 | 3 56 | 350 | 23 20 |
| 30 | 2 0 | 60 | 4 0 | 360 | 24 0 |

4207. The Uſe of this Table is eaſy by the Example following. Let it be required to find what Time is requi.ed for the Motion of 108° 4′ 45″ under the Meridian?

| | | | h | ′ | ″ |
|---|---|---|---|---|---|
| Then againſt | 100° | is | 6 | 40 | 0 |
| | 8° | —— —— | 0 | 32 | 0 |
| | 4′ | —— —— | 0 | 0 | 16 |
| | 45″ | —— —— | 0 | 0 | 3 |
| The Anſwer is | | —— | 7 | 12 | 19 (See 4074.) |

4208. By the ſecond Table you find the Motion in the Equator, or Equinoctial, correſponding to any given Time, as is plain by the Example.

## TABLE II.

*Shewing the* MOTION *correſponding to any given* TIME.

| Hours. | Degrees. | Min. Sec. Th. | Deg. Min. Min. Sec. Sec. Th. | Min. Sec. Th. | Deg. Min. Min. Sec. Sec. Th. |
|---|---|---|---|---|---|
| 1 | 15 | 1 | 0 15 | 31 | 7 45 |
| 2 | 30 | 2 | 0 30 | 32 | 8 0 |
| 3 | 45 | 3 | 0 45 | 33 | 8 15 |
| 4 | 60 | 4 | 1 0 | 34 | 8 30 |
| 5 | 75 | 5 | 1 15 | 35 | 8 45 |
| 6 | 90 | 6 | 1 30 | 36 | 9 0 |
| 7 | 105 | 7 | 1 45 | 37 | 9 15 |
| 8 | 120 | 8 | 2 0 | 38 | 9 30 |
| 9 | 135 | 9 | 2 15 | 39 | 9 45 |
| 10 | 150 | 10 | 2 30 | 40 | 10 0 |
| 11 | 165 | 11 | 2 45 | 41 | 10 15 |
| 12 | 180 | 12 | 3 0 | 42 | 10 30 |
| 13 | 195 | 13 | 3 15 | 43 | 10 45 |
| 14 | 210 | 14 | 3 30 | 44 | 11 0 |
| 15 | 225 | 15 | 3 45 | 45 | 11 15 |
| 16 | 240 | 16 | 4 0 | 46 | 11 30 |
| 17 | 255 | 17 | 4 15 | 47 | 11 45 |
| 18 | 270 | 18 | 4 30 | 48 | 12 0 |
| 19 | 285 | 19 | 4 45 | 49 | 12 15 |
| 20 | 300 | 20 | 5 0 | 50 | 12 30 |
| 21 | 315 | 21 | 5 15 | 51 | 12 45 |
| 22 | 330 | 22 | 5 30 | 52 | 13 0 |
| 23 | 345 | 23 | 5 45 | 53 | 13 15 |
| 24 | 360 | 24 | 6 0 | 54 | 13 30 |
| | | 25 | 6 15 | 55 | 13 45 |
| | | 26 | 6 30 | 56 | 14 0 |
| | | 27 | 6 45 | 57 | 14 15 |
| | | 28 | 7 0 | 58 | 14 30 |
| | | 29 | 7 15 | 59 | 14 45 |
| | | 30 | 7 30 | 60 | 15 0 |

4209. Let it be required to find what Motion in the Equator is performed in 6 Hours 20 Minutes, and 38 Seconds of Time.

| | | | ° | ′ | ″ |
|---|---|---|---|---|---|
| Then againſt | 6h | is | 90 | 0 | 0 |
| | 20′ | — | 5 | 0 | 0 |
| | 38″ | — | 0 | 9 | 30 |
| The Anſwer | | — | 95 | 9 | 30 |

(See 4102.)

4210. In Books of Geography you find TABLES of the LATITUDE and LONGITUDE of Places. By taking the Difference of Longitude between any two Places you find the Time correſponding thereto, and conſequently, if the Time be given in one Place, you know the Time in the other. For Example, if the Longitude of TORNEA in *Lapland* be 23° 48′ 45″ Eaſt of *London*; then by Table I. that Motion in Longitude is made in 1h 35′ 15″ of Time; and conſequently, when it is Noon with us, it is 35′ 15″ paſt One o'Clock there.

4211. In TABLES of ASTRONOMY, you find thoſe of the *right* ASCENSION and DECLINATION of the Stars; and alſo of the Sun for every Day in the Year. Now the Difference of the right Aſcenſion of the Sun, and any particular Star turned into Time, ſhews the Diſtance of Time at which the Sun and Star come upon the Meridian. For Example: On the 3d Day of *May*, the Sun's right Aſcenſion is 40° 56′ = 2h 43′ 44″, and in the preſent Age, that of the bright Star of *Aries* is 28° 23′ = 1h 53′ 13″; the Difference in Time is 50′ 31″, and ſo much ſooner will the Star be upon the Meridian than the Sun, or before Noon on that Day. The right Aſcenſion of ALDABARAN is 65° 30′ 3″ = 4h 21′ 17″ this Exceſs above that of the Sun, gives 2h 6′ 33″, and by that interval of Time it will be upon the Meredian after the Sun.

## CHAP. XIII.

*The* DESCRIPTION *and* USE *of a* TRANSIT GLOBE, *by which all the* PHASES *of the* TRANSIT *may be inſtantly exhibited for any given* TIME *and* PLACE, *by a new* Apparatus *for that Purpoſe.*

4212. HAVING mentioned a TRANSIT GLOBE, which I imagined would be much more ſerviceable than a MAP for facilitating the Ideas relative to the Nature and various *geographical Phænomena* of the Tranſit, and therefore as ſuch recommended it to the curious in the Aſtronomy and Geography of Tranſits; it may, perhaps, be neceſſary to give here a little further Account of the Conſtruction and Uſe of ſuch a Globe; which, after what has been already ſaid, will require but few Words.

4213. And in order that what I have to ſay may be better underſtood, I ſhall explain it by Fig. 6. Plate III. where P is the North Pole of the World, and the Center of the Circle G H I deſcribed about it at the Diſtance P G = 22° 27′ = Sun's Declination. The Globe being rectified for the *beginning and end of the Tranſit*, and the *two Circles* drawn as directed (4146). Then part of the firſt Circle is here denoted by E G Q at right Angles to the Meridian G K in which the Sun then is, *viz.* at the *beginning of the Tranſit*; and F H O repreſents part of the 2d Circle at right Angles to the Meridian H L, wherein the Sun is when the *Tranſit ends.*

4214. But beſides theſe two Circles there is upon the Tranſit Globe a third Circle, part whereof is ſhewn by the Arch M C N at right Angles to the Meridian C P I, which the Sun poſſeſſes at the *middle of the Tranſit.* Theſe three Circles divide the Surface of the Globe into two large Gores, and four ſmall ones; they are diſtinguiſhed from each on the ſmall *three-inch* pocket Globes by different

rent tranſparent Colours; but upon the larger Globe of twelve Inches Diameter, only by coloured Circles; and then by the Caſt of an Eye it will be ſeen.

*Firſt*, What Parts of the Earth are inhabited by thoſe who view the *whole Tranſit from beginning to end*, viz. thoſe which live in the Gore, E A B F marked with a *red Colour* upon the ſmall Globes.

*Secondly*, The other large Gore oppoſite to this is not coloured at all but appears *white*, in all which (except a little Triangle at the other End) *no part of the Tranſit will be viſible.*

*Thirdly*, The firſt ſmall Gore on the Weſt Side contain'd between the Circles E A and A M, ſhews all thoſe Places who ſee not the beginning of the Tranſit, but to whom the Sun will be ſeen to riſe with the Planet from the beginning at A E, to the middle of the Tranſit at A M.

*Fourthly*, In the ſecond ſmall Gore contiguous to the foregoing (as contained between the Circles M B and O B,) are contained all thoſe Parts of the Earth to whom the Sun will be ſeen to riſe with Venus upon its Disk from the *middle of the Tranſit* at M B, *to the end* in the Circle O B.

*Fifthly*, On the other (or Eaſtern) Side the Globe are the other two ſmall Gores; in the fartheſt of which, between the Circles A N and A Q, are contained all thoſe Parts of the World where the Sun will ſet with Venus upon its Disk from the *beginning to the middle.*

*Sixthly*, In the ſmall Gore between the Circles B N and B F, you will ſee all Parts of the Earth's Surface where the Sun ſets with the Planet upon its Disk, from the *middle of the Tranſit to the end.*

*Seventhly*, The ſmall triangular Space A B D, ſhews all thoſe Places who will ſee the *beginning and end of the Tranſit*, but will be gradually deprived of the other Phaſes from the beginning to the middle which they can ſee nothing at all of, the Triangle A D B C being then intirely under the Horizon M C N. After this, they recover Sight

of the Sun and the Tranſit by Degrees, till all obſerve it at the end.

*Eighthly*, At the farther End of the oppoſite Gore (at the South Pole) is ſuch another ſmall Triangle, and of an equal Area, the Inhabitants of which will gradually ſee the Tranſit from the beginning to the middle, which will be ſeen by them all; but after that, they will by Degrees looſe Sight of the Sun, till the end of the Tranſit becomes inviſible to them all.

4215. Theſe two Triangles are likewiſe diſtinguiſhed by proper Colours, as alſo each of thoſe *ſix Places* where the Tranſit will be obſerved to the greateſt Advantage. (See 4152, *&c.*) Therefore upon the TRANSIT GLOBE you will obſerve every Diſtinction ready made to your Hand, for forming the cleareſt and moſt accurate and pleaſing Ideas of the Whole, and every Part of this grand and moſt important of all celeſtial Phænomena.

4216. But what renders this Globe of the moſt general Uſe, is the *horary Quadrant of Altitude*, which by taking the Altitude of a certain Semicircle, ſhews the Phaſis of the Tranſit at any given Minute of the Paſſage to all the Inhabitants of the illumined Surface of the Earth. The Rationale of this *Apparatus* is founded in this Conſideration, that the Diſtance of the Semicircle E Q of the *beginning*, and the Semicircle O F of the end of the Tranſit, (or the Angle E D O) is equal to 86° 4′; and therefore if that Extent be laid down in a *Line of Hours* upon the Quadrant of Altitude, including the whole Duration of the Tranſit, then by it we can take the Elevation of the Semicircle E Q above the Weſtern Horizon of the illumined Disk, and ſolve the following general Problem, *viz.*

4217. It is required to rectify the Globe for ſhewing the Appearance of the Tranſit at 15′ paſt IX. (by the Time at LONDON). To do this, elevate the North Pole of the Globe 22° 27′ above the Horizon on the North Part, then fix the *horary Quadrant* to the Meridian in the Zenith, and bring

bring it to the Weſtern Part of the Horizon. Laſtly, elevate the Semicircle E Q to the given Hour and Minute upon the Quadrant, and there let the Globe reſt; and there will be now a general Exhibition of all the Parts of the Earth where the *Tranſit* at the given Moment can, or cannot, be ſeen; as alſo the Time of Day to each particular Place, and the Diſtance the Planet is advanced in the tranſit Line upon the Sun's Disk.

4218. All theſe Things will undoubtedly appear too eaſy to any Perſon acquainted with the Uſe of the common *terreſtrial Globe* to require any further Explication here, I ſhall conclude with only obſerving, that the ſame Globe (of 3 Inches Diameter) may be applied to an Orrery, ſo as to make a *tranſit* TELLURIAN, by which every thing relative to the Doctrine of *Parallaxes*, and the general *Phænomena* of the TRANSIT, may be exhibited in the moſt natural Manner poſſible.

4219. In Fig. 7. is a Repreſentation of the Tranſit at *Sun-ſet*, as it will be obſerved at LONDON, if the Weather be fine. A D E is the Semi-disk of the Sun, with the Center C in the Horizon. C B is the Poſition of the Ecliptic, making an Angle A C B of 39° ½; and D E is the Poſition of the *tranſit Line*, in which the Place of the Planet is at V at Sun-ſet.

4120. That there may be no Trouble in finding the ſix Places where Obſervations on the Tranſit may be made to the greateſt Advantage, I have here put them together in one View, *viz.*

| | | | | | |
|---|---|---|---|---|---|
| 1. The accelerated Ingreſs | Lat. | 51 | 28 | 17 | N. |
| | Long. | 0 | 6 | 12 | E. |
| 2. The retarded Ingreſs | Lat. | 43 | 30 | 33 | S. |
| | Long. | 162 | 49 | 15 | W. |
| 3. The retarded Egreſs | Lat. | 22 | 58 | 37 | N. |
| | Long. | 68 | 34 | 15 | E. |

4. The

| | | | | | |
|---|---|---|---|---|---|
| 4. The accelerated Egreſs | Lat. | 17 | 16 | 33 | S. |
| | Long. | 128 | 44 | 15 | W. |
| 5. The Duration, a *Maximum* | Lat. | 67 | 49 | 23 | N. |
| | Long. | 24 | 20 | 30 | E. |
| 6. The Duration, a *Minimum* | Lat. | 38 | 28 | 33 | S. |
| | Long. | 155 | 32 | 30 | W. |

END OF THE SECOND PART.

Plate 1.

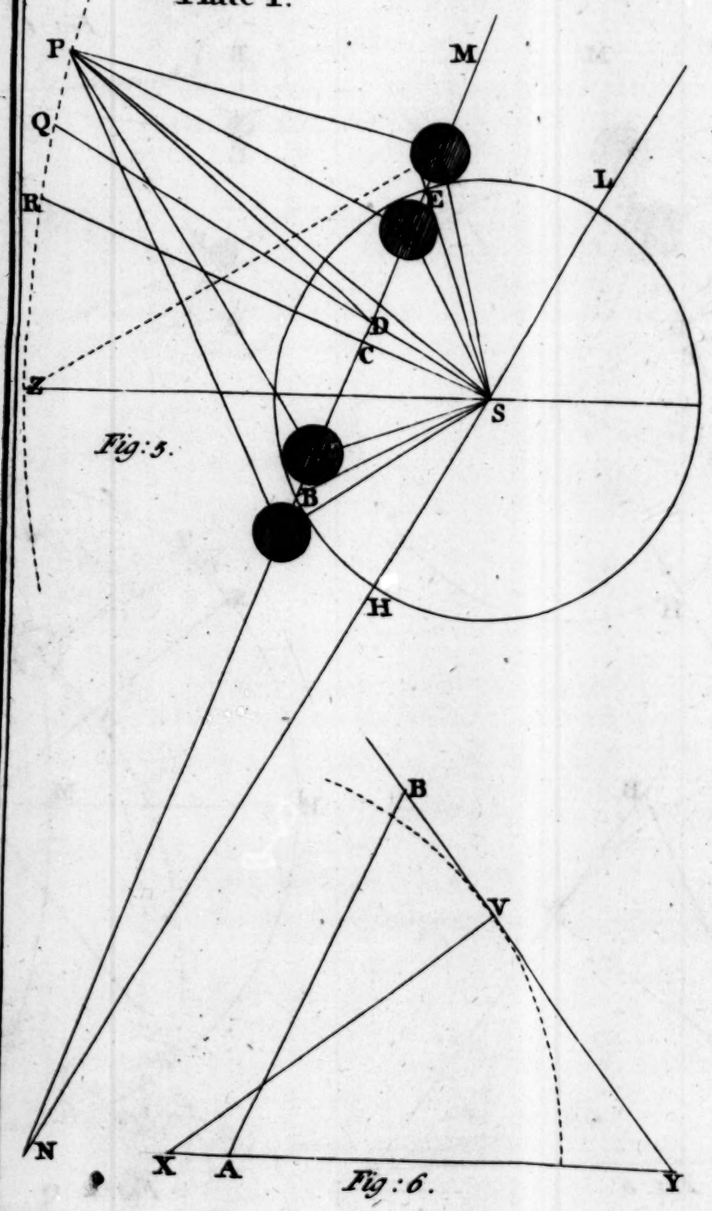



Fig: 5.

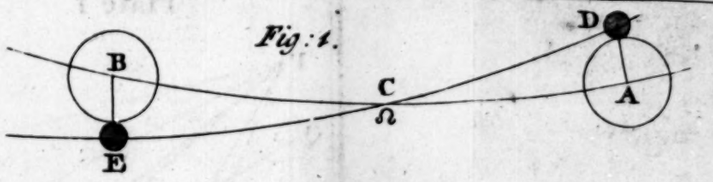



Fig: 1.

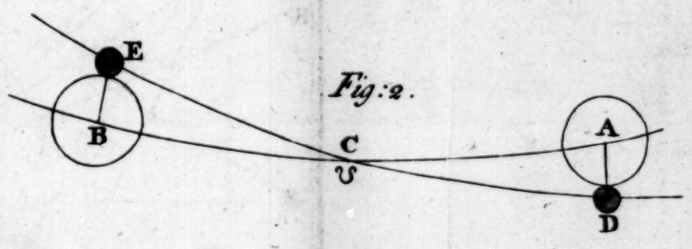



Fig: 2.

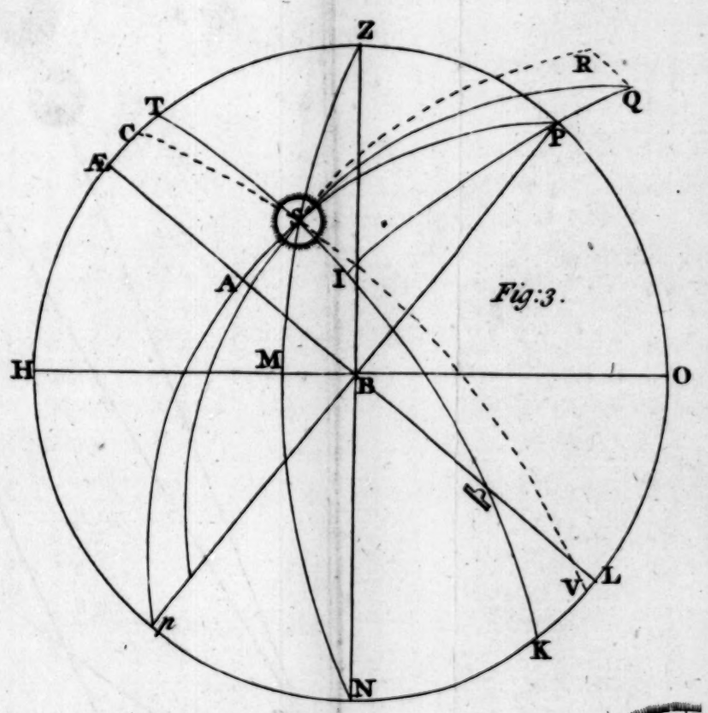



Fig: 3.




Fig: 6.

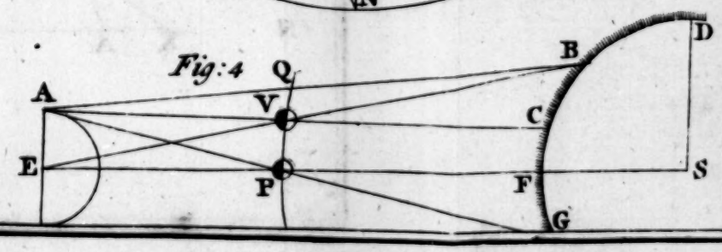



Fig: 4

Plate. II.

*Egress retarded by the Sum v X + v Y.*

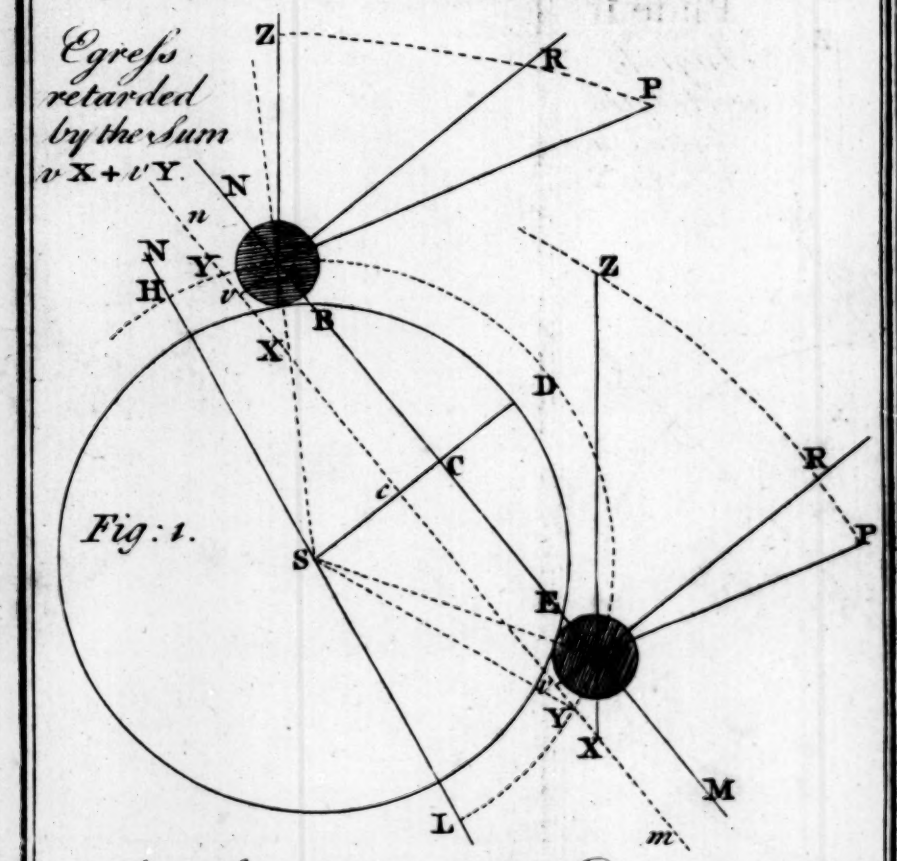



*Fig: 1.*

*Ingress retarded by the Diff: v X - v Y.*

*Ingress accelerated by the Sum v X + v Y.*

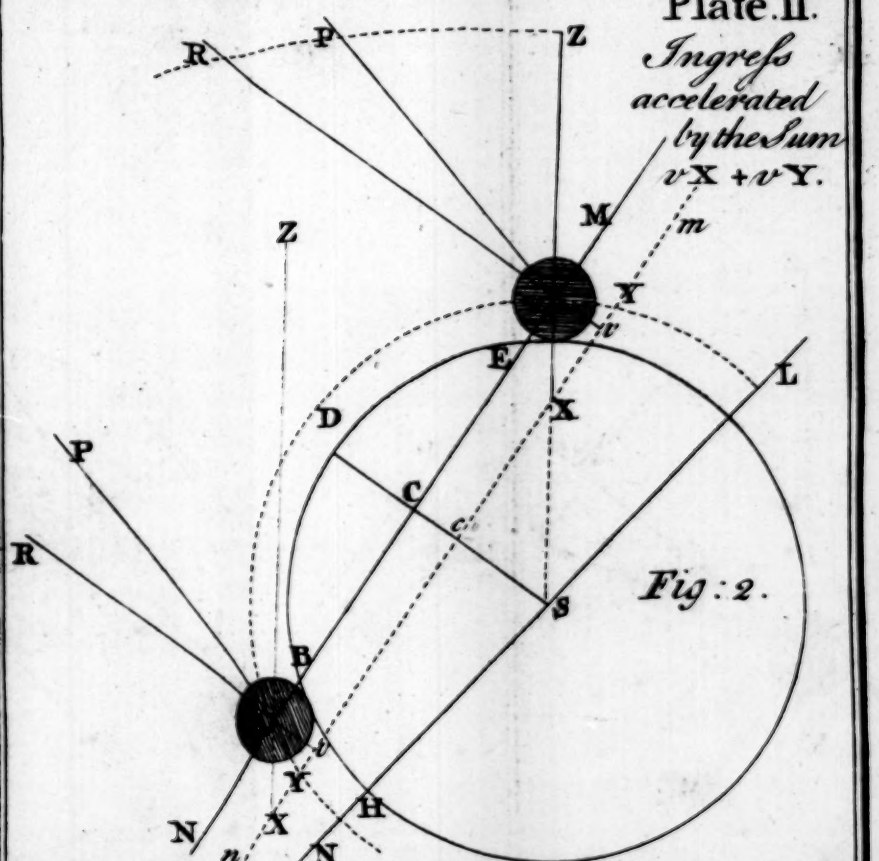



*Fig: 2.*

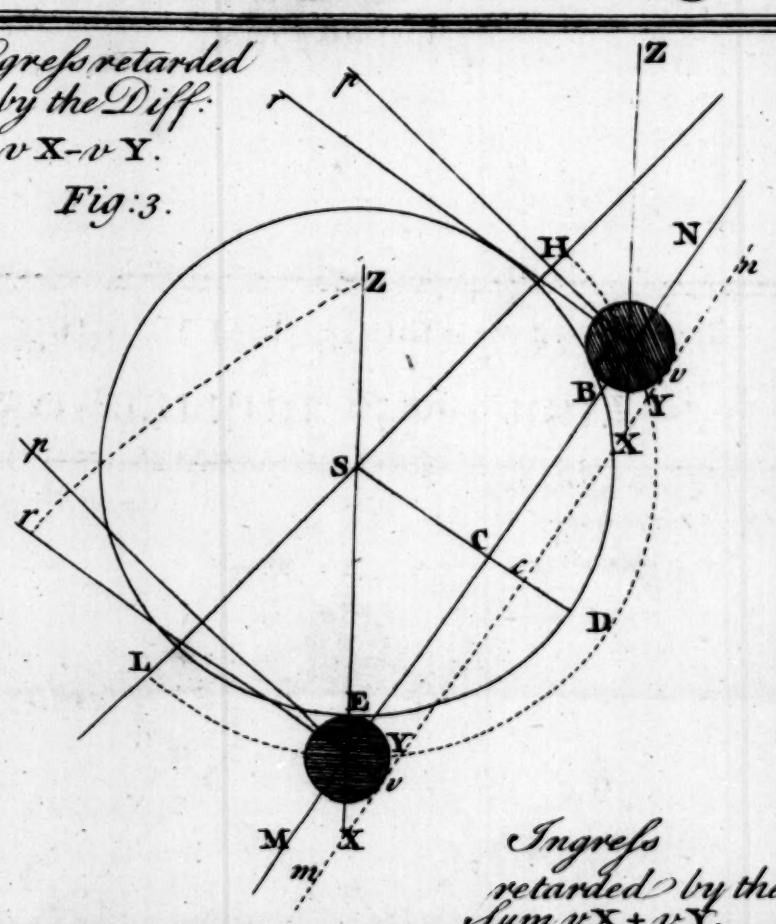
*Egress accelerated by the Diff: v X - v Y.*

**NORTH SIDE of the ORBIT of VENUS.**
**Eastern Hemisphere or Morning.**

**NORTH SIDE of the ORBIT of ♀**
**Western Hemisphere or Afternoon.**

*Egress retarded by the Diff: v X - v Y.*

*Fig: 3.*




*Ingress retarded by the Sum v X + v Y.*

*Ingress accelerated by the Diff: v X - v Y.*

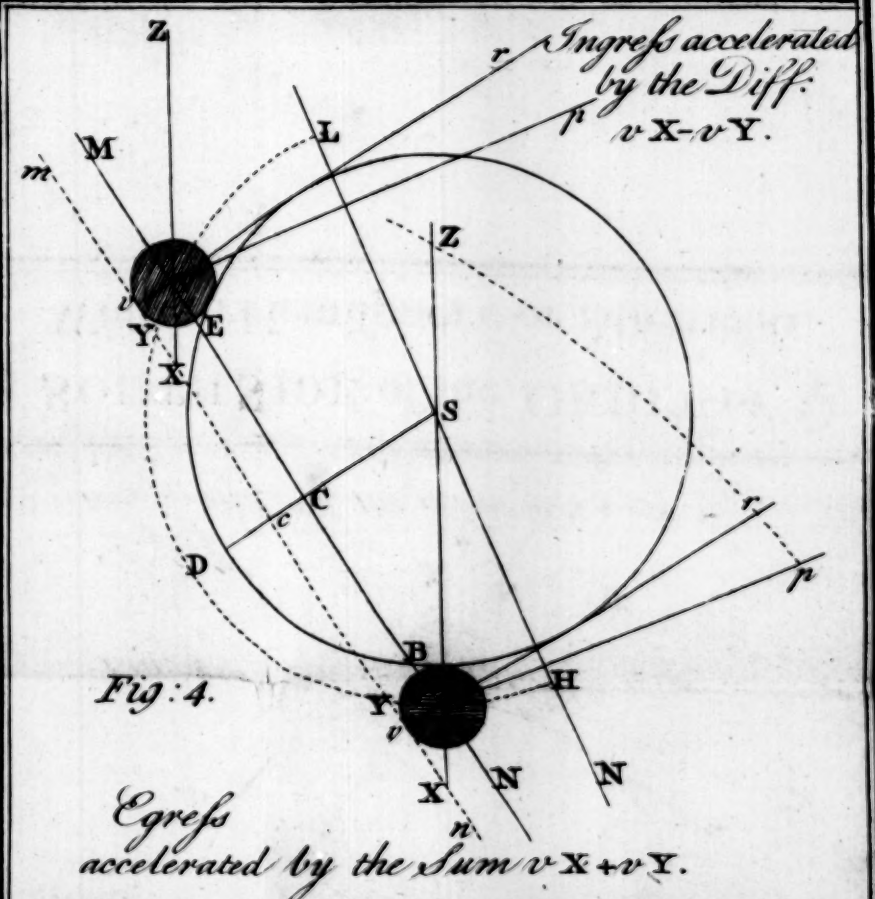


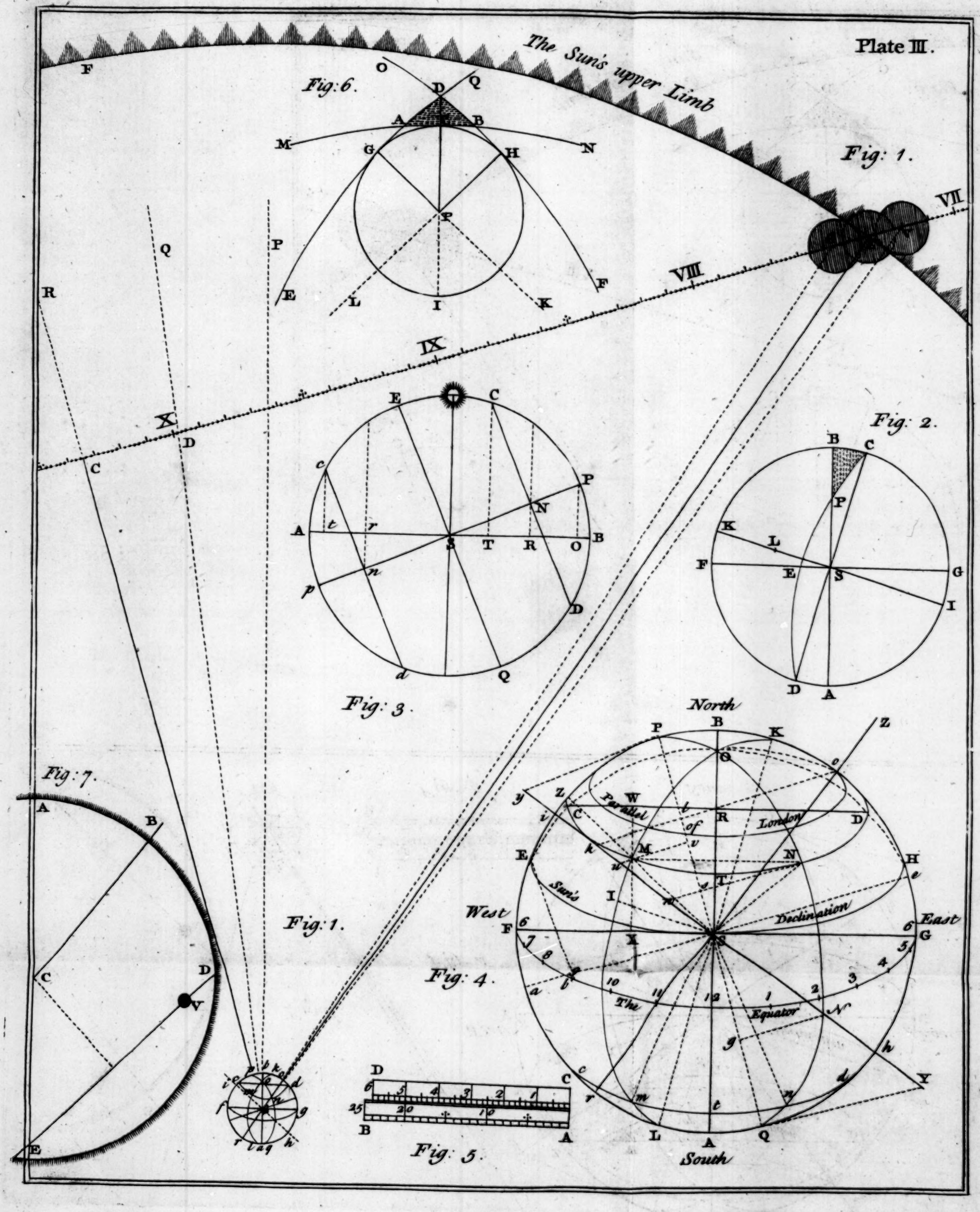

*Fig: 4.*

*Egress accelerated by the Sum v X + v Y.*

**SOUTH SIDE of the ORBIT of ♀**
**Eastern Hemisphere or Morning.**

**SOUTH SIDE of the ORBIT of ♀**
**Western Hemisphere or Afternoon.**

Plate III.


The Sun's upper Limb
Fig: 6.
Fig: 1.
Fig: 7.
Fig: 1.
Fig: 3.
Fig: 2.
Fig: 4.
Fig: 5.
North
South
West
East
Parallel of London
Sun's Declination
Equator